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Computer Science 4252: Introduction to Computational Learning Theory
Problem Set #4 Spring 2006

Due 5:00pm Monday, Mar 27, 2005

Problem 1 Show that there is a domain X such that for any integer $d > 0$ there is a concept class C over X of VC dimension d such that for any $m > 0$ there is a set $S \subset X$ of m points such that $|\Pi_C(S)| = \Phi_d(m)$.

Problem 2
(a) Let C_1 be the class of unions of k intervals over the line. Determine the exact value of the VC dimension of C_1.

(b) Now let C_2 be the class of axis parallel rectangles (i.e. boxes) in $\Re^n$. (For example, the concept $c = \{(x, y, z) : 1 \leq x \leq 3, -4 \leq y \leq -2 \text{ and } 5 \leq z \leq 6\}$ is an example of an axis parallel rectangle in $\Re^3$.) Determine the exact value of the VC dimension of C_2.

Problem 3 Recall that a parity function is a function which tests the parity of some subset $S \subseteq \{x_1, \ldots, x_n\}$ of the input variables. The value of $f_S(x)$ is 1 if an odd number of the variables in S have value 1 and is 0 otherwise. The class of parity functions is $\mathcal{P} = \{f \mid f = \oplus_{x_i \in S} x_i, \text{ where } S \subseteq \{x_1, \ldots, x_n\}\}$.
Prove that any PAC learning algorithm for the class of parity functions over variables $x_1, \ldots, x_n$ must require $\Omega(n/\epsilon)$ examples.

Problem 4 A concept class C over a domain X is said to be *linearly ordered* if (i) C contains at least two concepts; and (ii) for any $c_1, c_2 \in C$ either $c_1 \subseteq c_2$ or $c_2 \subseteq c_1$.

(i) Show that if C is linearly ordered then the VC dimension of C is 1.

(ii) Show that if the VC dimension of C is 1 and $\emptyset \in C$ and $X \in C$ then C is linearly ordered.

(iii) Let $H_1, H_2, \ldots, H_s$ be a sequence of concept classes, each of which is linearly ordered over X, and let $H = \{h_1 \cup \ldots \cup h_s \mid h_i \in H_i\}$. Show that the VC dimension of H is at most s. **Hint:** How many size-1 subsets of a set S can be induced by intersecting with H?

Problem 5 A set S of points in Euclidean space is said to be *convex* if the line segment joining any pair of points of S lies entirely in S (i.e. for all $x \in S, y \in S$ and all $0 \leq \gamma \leq 1$, the point $\gamma x + (1 - \gamma)y$ is also in S). Prove that the class of all convex subsets of $[0, 1]^2$ has infinite VC dimension.