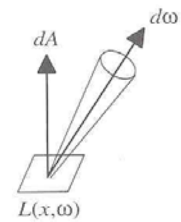
Computer Graphics (Fall 2008)

COMS 4160, Lecture 19: Illumination and Shading 2

http://www.cs.columbia.edu/~cs4160

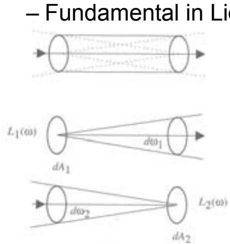
Radiance

- Power per unit projected area perpendicular to the ray per unit solid angle in the direction of the ray
- Symbol: $L(x,\omega)$ ($W/m^2\ sr$)
- Flux given by
  $d\Phi = L(x,\omega) \cos\theta \, d\omega \, dA$



Radiance properties

- Radiance is constant as it propagates along ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.



$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

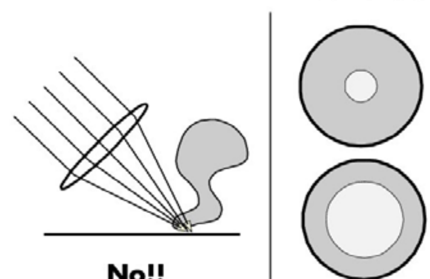
$$d\omega_1 = dA_2 / r^2 \qquad d\omega_2 = dA_1 / r^2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Quiz

Does radiance increase under a magnifying glass?



No!!

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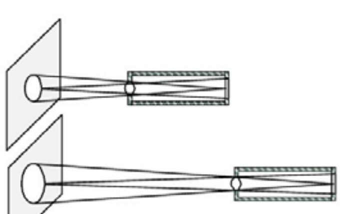
Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?



No!!

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Radiance properties

- Sensor response proportional to radiance (constant of proportionality is throughput)
  - Far away surface: See more, but subtends smaller angle
  - Wall equally bright across viewing distances

Consequences

- Radiance associated with rays in a ray tracer
- Other radiometric quants derived from radiance

Irradiance, Radiosity

- Irradiance E is radiant power per unit area
- Integrate incoming radiance over hemisphere
  - Projected solid angle ($\cos\theta \, d\omega$)
  - Uniform illumination: Irradiance = π [CW 24,25]
  - Units: W/m^2
- Radiosity
  - Power per unit area leaving surface (like irradiance)

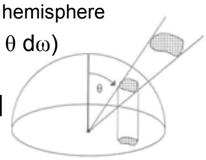


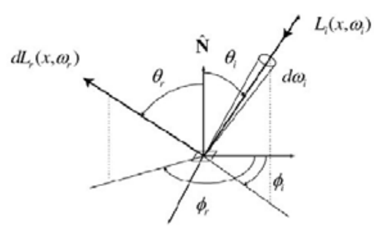
Figure 2.8: Projection of differential area.

Building up the BRDF

- Bi-Directional Reflectance Distribution Function [Nicodemus 77]
- Function based on incident, view direction
- Relates incoming light energy to outgoing light energy
- We have already seen special cases: Lambertian, Phong
- In this lecture, we study all this abstractly

The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

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BRDF

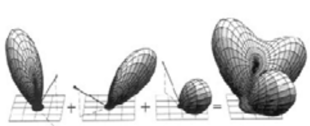
- Reflected Radiance proportional to Irradiance
- Constant proportionality: BRDF [CW pp 28,29]
  - Ratio of outgoing light (radiance) to incoming light (irradiance)
  - Bidirectional Reflection Distribution Function
  - (4 Vars) units 1/sr

$$f(\omega_i, \omega_r) = \frac{L_r(\omega_r)}{L_i(\omega_i)\cos\theta_i d\omega_i}$$

$$L_r(\omega_r) = L_i(\omega_i) f(\omega_i, \omega_r) \cos\theta_i d\omega_i$$

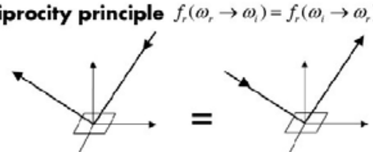
Properties of BRDF's

1. Linear



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle $f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$



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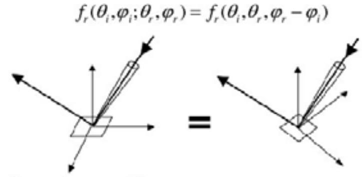
Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i, \varphi_i; \theta_r, \varphi_r) = f_r(\theta_i, \theta_r, \varphi_r - \varphi_i)$$



Reciprocity *and* isotropy

$$f_r(\theta_i, \theta_r, \varphi_r - \varphi_i) = f_r(\theta_r, \theta_i, \varphi_i - \varphi_r) = f_r(\theta_i, \theta_r, |\varphi_r - \varphi_i|)$$

4. Energy conservation

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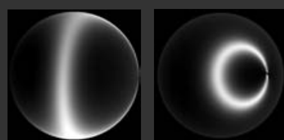
Isotropic vs Anisotropic

- Isotropic: Most materials (you can rotate about normal without changing reflections)
- Anisotropic: brushed metal etc. preferred tangential direction

Isotropic

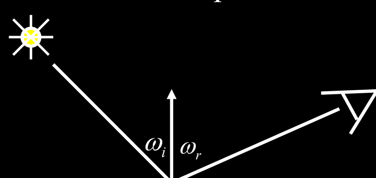


Anisotropic

Radiometry

- Physical measurement of electromagnetic energy
- We consider light field
  - Radiance, Irradiance
  - Reflection functions: Bi-Directional Reflectance Distribution Function or BRDF
  - *Reflection Equation*
  - Simple BRDF models

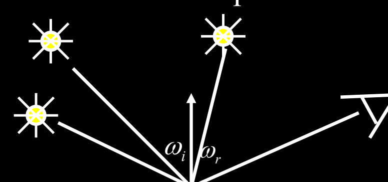
Reflection Equation



$$L_r(\omega_r) = L_i(\omega_i)\, f(\omega_i, \omega_r)(\omega_i \bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Reflection Equation

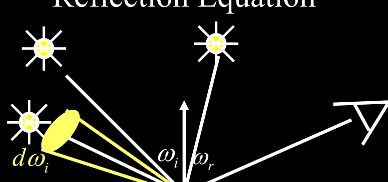


Sum over all light sources

$$L_r(\omega_r) = \sum_i L_i(\omega_i)\, f(\omega_i, \omega_r)(\omega_i \bullet n)$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Reflection Equation



Replace sum with integral

$$L_r(\omega_r) = \int_\Omega L_i(\omega_i)\, f(\omega_i, \omega_r)(\omega_i \bullet n)\, d\omega_i$$

Reflected Radiance (Output Image) | Incident radiance (from light source) | BRDF | Cosine of Incident angle

Radiometry

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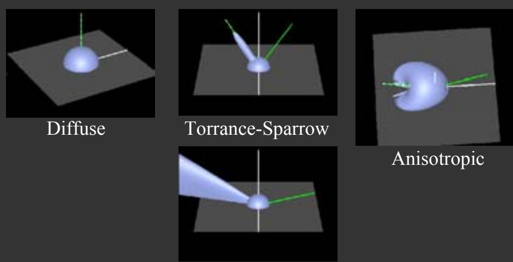
Brdf Viewer plots



bv written by Szymon Rusinkiewicz

Demo

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).

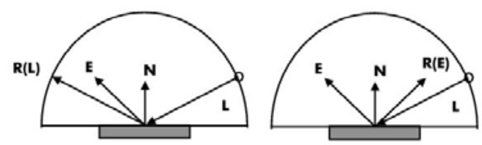
$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos\theta_i \, d\omega_i = f_{r,d} \int L_i(\omega_i) \cos\theta_i \, d\omega_i = f_{r,d} E$$

$$M = \int L_r(\omega_r) \cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

Phong Model



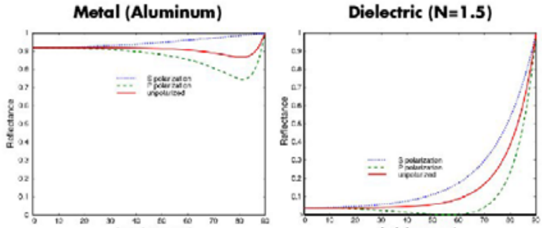
$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

Distributed light source!

Fresnel Reflectance

Metal (Aluminum) **Dielectric (N=1.5)**



Gold F(0)=0.82
Silver F(0)=0.95

Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

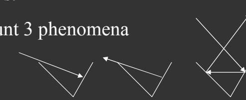
Analytical BRDF: TS example

- One famous analytically derived BRDF is the Torrance-Sparrow model.
- T-S is used to model specular surface, like the Phong model.
  - more accurate than Phong
  - has more parameters that can be set to match different materials
  - derived based on assumptions of underlying geometry. (instead of ‘because it works well’)

Torrance-Sparrow

- Assume the surface is made up grooves at the microscopic level.

- Assume the faces of these grooves (called microfacets) are perfect reflectors.
- Take into account 3 phenomena



Torrance-Sparrow Result

Fresnel term: allows for wavelength dependency

Geometric Attenuation: reduces the output based on the amount of shadowing or masking that occurs.

$$f = \frac{F(\theta_i)G(\omega_i,\omega_r)D(\theta_h)}{4\cos(\theta_i)\cos(\theta_r)}$$

Distribution: distribution function determines what percentage of microfacets are oriented to reflect in the viewer direction.

How much of the macroscopic surface is visible to the light source

How much of the macroscopic surface is visible to the viewer

Other BRDF models

- Empirical: Measure and build a 4D table
- Anisotropic models for hair, brushed steel
- Cartoon shaders, funky BRDFs
- Capturing spatial variation
- Very active area of research

Complex Lighting

- So far we’ve looked at simple, discrete light sources.
- Real environments contribute many colors of light from many directions.
- The complex lighting of a scene can be captured in an Environment map.
  - Just paint the environment on a sphere.

Environment Maps

- Instead of determining the lighting direction by knowing what lights exist, determine what light exists by knowing the lighting direction.

Blinn and Newell 1976, Miller and Hoffman, 1984
Later, Greene 86, Cabral et al. 87

Demo

Conclusion

- All this (OpenGL, physically based) are local illumination and shading models
- Good lighting, BRDFs produce convincing results
  - Matrix movies, modern realistic computer graphics
- Do not consider global effects like shadows, interreflections (from one surface on another)
  - Subject of next unit (global illumination)

What's Next

- Have finished basic material for the class
  - Texture mapping lecture later today
- Review of illumination and Shading
- Remaining topics are global illumination (written assignment 2): Lectures on rendering eq, radiosity
- Historical movie: Story of Computer Graphics
- Likely to finish these by Dec 1: No class Dec 8,
- Work instead on HW 4, written assignments
- Dec 10? will be demo session for HW 4