Remarks on analyzing certain riddles

Consider the first of Smullyan's Riddles seen in class:

GOLD	SILVER	LEAD
"the portrait is in this casket"	"the portrait is not in this casket"	"the portrait is not in the gold casket"

If you know that

1. at most one of the statements is true, and
2. the portrait is in one (and only one) of the caskets

then which one is it in?

Let's start by transcribing the facts given in to a more symbolic form. Let

G	be the statement	"The portrait's in the gold casket"
S	be the statement	"The portrait's in the silver casket"
L	be the statement	"The portrait's in the lead casket"

Then we can restate the problem as follows:

GOLD	SILVER	LEAD
G	$\neg S$	$\neg G$

You are also given that no more than one statement is true, i.e. *no two of them are both true*. This can be stated via the following three axioms:

A1 $\neg(G \wedge \neg S)$ and

A2 $\neg(G \wedge \neg G)$ and

A3 $\neg(\neg S \wedge \neg G)$

We also have that:

A4 $G \vee S \vee L$ (the portrait is in one of the caskets) and

A5 $G \rightarrow (\neg S \wedge \neg L)$ and

A6 $S \rightarrow (\neg G \wedge \neg L)$ and

A7 $L \rightarrow (\neg G \wedge \neg s)$

(i.e., if the portrait's in the gold casket it can't be in the silver and it can't be in the lead, etc..

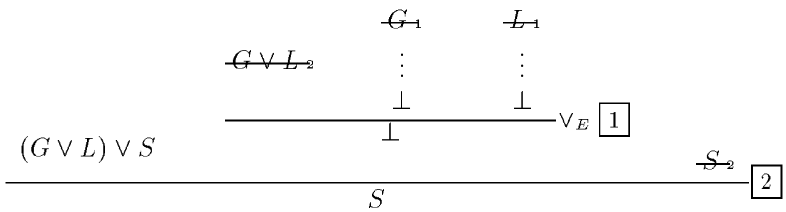
Now derive one of the following: S, G or L.

The basic strategy is exhaustion of all cases, which may take the following form. Show that *using the axioms A1-7*

if G then contradiction
if L then contradiction

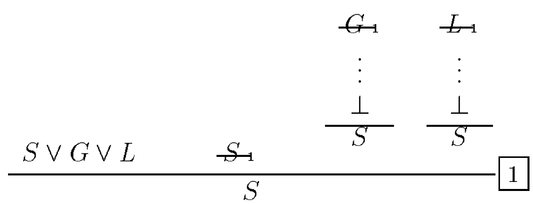
hence, given $G \vee S \vee L$, we must have S.

There are a number of ways to transcribe this to a natural deduction proof. Here's one strategy. First observe that $G \vee S \vee L$ isn't officially a real proposition. One must either write $G \vee (S \vee L)$ or $(G \vee S) \vee L$, which is logically equivalent to it. In fact, we could have written the equivalent $(G \vee L) \vee S$ which is the one we'll use.



In other words we conclude S in all cases by two uses of $\vee_E$. The proofs that G and L lead to contradictions are missing. They will be supplied in class. (You might try to supply them too.)

Note that a convenient way of expressing the main idea would be to invoke some kind of 3-cases rule that looked like this:



No such rule exists, and it is clearly redundant (well, actually you have to show this in the homework), meaning that we don't need such a "special rule" to do logic. We already obtained the conclusion above by taking the cases two at a time. But this "pseudo-rule" seems to summarize the argument a little more clearly.