CS W3134: Data Structures in Java

Lecture #21: Graphs I
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Administrivia

- Alternate exam time?

Agenda

- Finish heaps
  - Let's look at the book's code briefly
- Graphs
  - Last data structure

What are graphs?

- Linked list :: trees → trees :: graphs
- In other words, we no longer limit the number of children each node may have, and we don't forbid loops (sometimes!)
- Examples?
  - Bridges of Konigsburg (p. 619)
    - Solution: vertices of odd degree make it impossible
    - Foundation of graph theory (1736)

Definitions

- Adjacency
- Path
  - Multiple definitions ☹
- Connected graph
- Directed graph
- Weighted graph
  - These two come later!

Representing a graph

- The OO way
- The canonical (and book) way
  - Adjacency matrix
    - I lied – we *will* use 2D matrices
  - Adjacency list
- Advantages and disadvantages?
- Book => separate vertex class
- For some reason, the book does it the latter

Searching graphs?

- Goal: find connectivity
- Depth-first search
  - Push node on a stack
  - While stack not empty:
    - Peek and get an unvisited adjacent node
    - Visit it (pushing it on the stack)
    - If no adjacent nodes, pop and repeat
  - Game searching and branching factor
- Breadth-first search
  - Same process, but queue instead

Complexity of BFS and DFS?

- Optimally, $O(V+E)$ – we visit every vertex a constant number of times and potentially travel every edge a constant number of times
- But this is only for an adjacency list; in an adjacency matrix version, it's $O(V^2)$ – we scan every row and every column in the adjacency matrix once
- Admittedly inefficient, but we knew that

Minimum spanning trees

- A (minimum) spanning tree is a subgraph with no cycles
  - Different in weighted graphs
- Remove graph redundancy
- Useful for many applications
  - Ex: minimize wiring
- In a minimum spanning tree, #E = #V – 1

Computing a MST

- Simple algorithm (p. 644): DFS and record the edges traveled
  - Don't worry about backtracking
  - Can also use BFS…

Next time

- Directed graphs