CS W3134: Data Structures in Java

Lecture #22: Graphs II
11/30/04
Janak J Parekh

Administrivia

- Alternate exam time?
- HW#5 due now
- HW#6 out today
  - Will be due Monday, 5pm right before reading week begins
  - Let's talk about the programming now

Agenda

- Graphs cont'd.

Directed graphs

- As earlier mentioned, useful for situations where we need to model "one-way" information
  - Streets
  - Trees are a subclass of directed graphs
  - Book: course prerequisites

Topological sort

- Come up with a legitimate ordering of processing the nodes
  - Often useful for *partial ordering* problems, such as aforementioned course prerequisites
  - Result: a order where no vertex y comes before a vertex x where x→y
  - There can be multiple correct answers!

Topological sort (II)

- Find a vertex that has no successors, i.e., arrows that point to *it*
  - Look at columns of the adjacency matrix
- Delete that vertex and print it out
- Repeat
- What kinds of graphs doesn't this work for?
  - Cycles – what happens?
  - "Catch-22" in real life
  - In other words, works on generalized trees (multiple roots, etc.) – *DAG*

Topological sort (III)

- Complexity again $O(V+E)/O(V^2)$
- How to find node with no successors?
- How do you delete a node?

Connectivity in directed graphs

- Can't just do an arbitrary BFS or DFS
  - Connectivity *depends* on starting node, i.e., "what can you reach from node X?"
  - Do DFS from every vertex!
- Alternative: develop *connectivity matrix* from adjacency matrix
  - *Transitive closure* of adjacency matrix
  - If L → M and M → N, L → N

Warshall's Algorithm

- For all rows *y*,
  - For all columns *x* in row *y*,
    - If any value (x,y) is 1, then for all rows *z* in column *y*,
      - If (y,z) is 1, then (x,z) should be 1
- i.e., "transitive closure"

Warshall's Algorithm (II)

- That's it!
  - Remember array references are "backwards" [y][x]
- Yes, this actually works in one pass – all the holes are filled
- What's the complexity of *this* algorithm?

Weighted graphs

- How to represent? Not just 0s and 1s in the adjacency matrix; weight instead
- Example
  - Roadmap!
- Can be directed or undirected

Next time

- Continue weighted graphs
- We're almost there. ☺