Project 4: Olympic Road Race

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1 Introduction

The Olympic road race mimics the actual Olympic road race where the goal of each team is to win a medal at the end of the race, either a Gold, Silver or Bronze. The nature of the problem is much more competitive than previous projects. We initially implemented a very mathematical solution for the problem using just the players on our team. The problem however, is that in a multi player environment there are many factors that come into play that interfere with our strategy. Therefore, we also developed some counter strategies.

The remainder of this document will discuss the strategies and counter-strategies we developed through the evolution of our player.

2 Strategies

This section will discuss the strategies and counter-strategies we considered and implemented for our final player as well as their advantages and shortcomings.

2.1 Traveling Max Speed

This is a very simple strategy developed during the very early stages of the project. From the project description we knew that given the default parameters, each player could go at a speed of about 9+ m/s without taking advantage of any drafting and finish the race with some energy left over. Hence, we decided to implement a player that would travel at this default speed for most of the time. Then at every turn we would recalculate the speed at which the player would make it to the finish line by using up all of its energy. The basic formula we used for this calculation is as follows:

$$v = \left(\frac{E}{D}\right)^{\frac{2}{3}} \tag{1}$$

where E is the energy the rider has at any point in time, D is the distance left from the current position to the finish line and v is the velocity the rider can go to finish the race using all of its energy.

Once v is calculated we do a simple check to see if the velocity calculated is greater than the current velocity we are traveling at. If this check is passed then we accelerate until we reach this value. This will ensure that our energy will not be going to waste when we reach the finish line.

2.2 Using Drafting

The major drawback of the strategy presented in section 2.1 is that it does not take advantage of drafting other players. In order to improve the above strategies we implemented a strategy where each rider would try to follow other riders that are ahead of them and are closest to them. For every turn, we calculate if there is a lane that consists of a player that is ahead of the current player. If so, then we change into that lane in an attempt to follow that player. Once we have done this we accelerate until either we are right behind the rider we are trying to follow or our calculation from equation 1 does not permit us to go any faster. Additionally, we will change into an empty lane and try to finish the race before the rider we are following if our calculation from equation 1 is greater than the speed of the person we are following.

This strategy is only useful in multi player mode as it requires other players we need to take advantage of. Although this strategy may work well in theory in practice it did not perform well due to large fluctuations in speed. Since the consumption of energy is exponential $v^2.5$, we are expending too much energy trying to catch up to faster riders causing us to go slower during later stages of the race. It is a much better strategy to travel at a constant speed due to this exponential factor. Furthermore, a more sophisticated method of choosing a rider to follow needs to be developed as the rider in front of us may not justify a smart player but could be dumb player going at fast speeds at beginning stages of the game and then dying out in half way through the race.

2.3 The Optimal Speed

Due to the major drawbacks in the drafting strategy presented, we decided to go back to the drawing board and use a more mathematical approach that takes into account drafting as well as teamwork within the riders in our team. The key thing to note is that not all players need to finish the race, only one rider from the team needs to finish the race in order to secure a medal. So if all riders in the team were to ride in a single line one after the other, then we can calculate a constant speed that will enable only our last rider to finish the race while the others in the front of the line will die out. This will enable us to go at a much faster rate since only the last player needs to finish the race and it is taking advantage of the wind resistance of the player in front of it until its death. Each player n is taking advantage of the player in position $n-1$ except for the player at the front of the line. Consider the following sets of equations:

$$v_1 = \left(\frac{E}{D}\right)^{\frac{2}{3}} \qquad v_2 = \left(.3\frac{E}{D}\right)^{\frac{2}{3}} \qquad v_3 = \left(.3^2\frac{E}{D}\right)^{\frac{2}{3}} \tag{2}$$

where v_n presents the velocity the line of riders should travel when they are exactly 2 meters apart and there are n riders. E is energy of the riders and D is the distance left to the end of race. We can create a more general equation as follows:

$$v_n = \left[\left(\sum_{i=1}^{n} f(d_i)^{i-1} \right) \left(\frac{E}{D} \right) \right]^{\frac{2}{3}} \qquad f(d) = \frac{5-d}{10} \tag{3}$$

where n represents the number of riders in a line, d_i is the distance between rider i and rider $i-1$, E is the energy left, D is the distance to the finish line and $f(d)$ defines the savings in energy from drafting. The speed obtained from this equation is the optimal speed that the riders should travel in order to finish the last rider with the best time.

Of course the major drawback in this approach is that the players do not begin the race in a line, as a result we must account for the time it takes for the players to form a line and then we can use equation 3 to solve for the speed at which the riders should travel. Figure 1 shows the basic essence of this strategy.

2.4 Forming a Line

Once the players have formed a line we can use equation 3 in order to calculate the optimal velocity and simply travel at that speed until the end of the race. There were several strategies that were considered when forming a line. This section will describe a few of them.

2.4.1 Staying Put

A primary concern when forming a line was to expend as little energy as possible since a greater E value will result in a higher velocity. In order to accomplish this goal, we implemented a strategy in which the riders take turns moving and shifting lanes. We first define a lane in which all riders should form a line, this lane number is computed using the average of the lane numbers of the positions of the furthest left and furthest right riders at the beginning of the race.

Once the lane number has been determined, each rider i will move $(n - i) \times 2$ meters and then stops. Note that the last player will not move preserving all of its energy. Then each player will change lanes until they reach the lane number computed above. Once all players have reached the desired lane, equation 3 is used to compute the velocity that is needed to travel. We take a conservative approach and use E and D for the first player. The riders can then accelerate and travel at that speed until they finish the race.

This strategy requires riders to fluctuate their speeds which causes more energy expenditure than need be. The stopping and regaining speed is especially counter productive when trying to save energy. Furthermore, this strategy we get varying results in energy levels and our conservative approach results in having the last rider finish with some energy remaining, which can be used to go at a greater speed.

2.4.2 Moving and Changing

The major drawback of using the Staying Put strategy to form lines in that some riders will need to speed up to a certain speed, then stop and then speed up again. This, effect

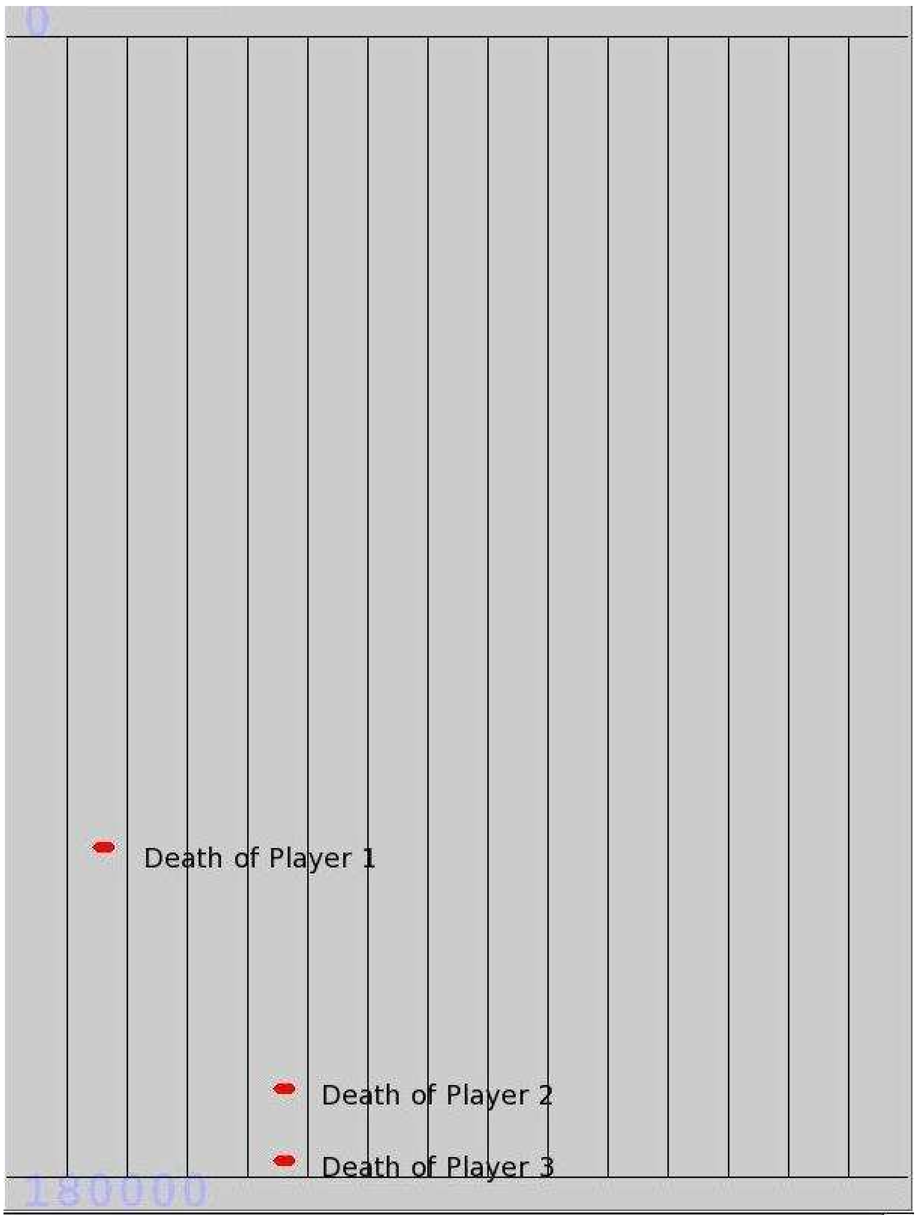

Figure 1: Traveling at optimal speed. Player 4 finishes the race

can be minimized by noticing that each rider must reach a certain speed when they are in line. As a result, each rider can start accelerating and in the process of attaining optimal speed they can come together to form a line.

Equation 3 is used to compute the optimal speed during the start of the race. Each player then accelerates till the player reaches $v_{optimal} - 2$ where 2 represents the minimum distance that can be alloted between each rider. This ensures maximum energy savings when drafting another player. Each player in turn then accelerates to $v_{optimal}$. When rider r_1 has reached the optimal speed, rider r_2 accelerates to $v_{optimal}$, then r_3 and so on, until everyone has reached $v_{optimal}$. This process of taking turns to accelerate from $v_{optimal} - 2$ to $v_{optimal}$ ensures that each rider has a 2 meter gap between them assuming that the acceleration per turn is $1m/s$. Each rider also changes until the desired lane to meet is reached and the line is formed. Once the line has been formed however, $v_{optimal}$ is recalculated to take into account the energy loss during line forming.

2.4.3 Handling Interferences

The aforementioned strategy does not work well when dealing with many other players in the race. This is because some players have a counter strategy such as blocking and other players are also changing lanes, interfering with our riders. As a result, our riders end up being more than 2 meters apart or even worse are not able to form a line. As a result, we needed some strategy to take of these obscure cases.

In order to handle these cases we started changing lanes before most other players in the race so that there are less interferences. So instead of starting the change of lane process when we reach $v_{optimal} - 2$ riders start the process when they reach $max(v_{optimal} - 5, 1.0)$. We use a $max()$ function here in cases where $\frac{E}{D}$ is small resulting in small values of $v_{optimal}$. We also account for the cases when riders are in the same line and they are more than 2 meters apart. Each rider that is more than 2 meters apart will accelerate till they are at least 2 meters from the rider in front of them, with the exception for the first rider in the front of the line.

This strategy of getting into line helped us in the mini-tournaments run in class as we were able to form a line particularly quickly in situations where are not as many interferences with other players. Figure 2 shows the ideal situation where our line forming worked particularly well (more open spaces and less players around unlike Figure 3).

2.5 Other Considerations

In addition to calculating the optimal speed and changing lanes in an efficient manner, we also needed to consider the strategies of other players. Some players were purely destructive that relied on blocking strategies. Other players attempted to draft riders from another team. In order to handle these situations, riders that have already formed a line will move randomly in either direction as well as try to move away from players that are around them. This method of constantly changing lanes is useful as we are

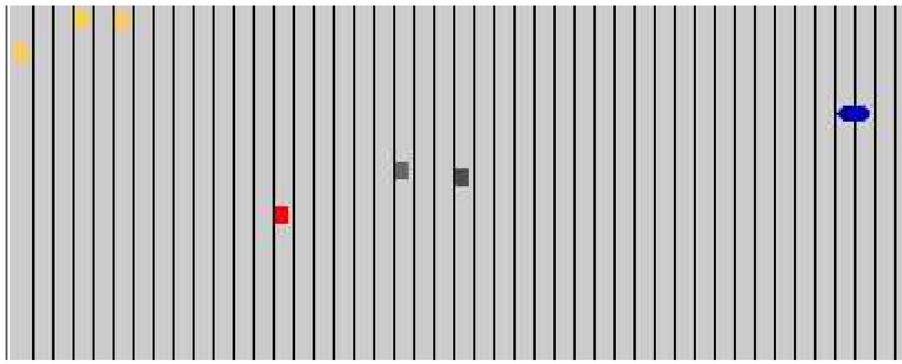

Figure 2: Widely spaced area makes it suitable for line forming

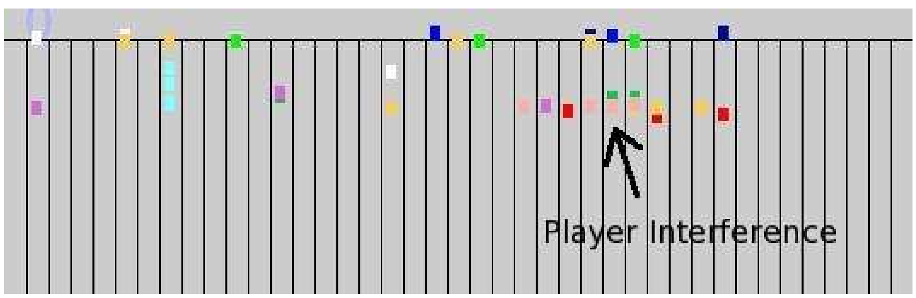

Figure 3: Interfering players when trying to get in line formation

trying to move away from players that may block us. Furthermore, random movement makes it difficult for other teams to draft the last player in our team.

We also considered strategies in which our line of players would draft other players. However, we decided that the marginal benefit from doing this was not great as it required fluctuations in speed in order to get behind another player. Additionally if other players are employing strategies similar to random movement to avoid being drafted there is only a $\frac{1}{3}$ we will draft someone else in a single turn. We decided this would not benefit us in the long run.

We noticed that our final rider was finishing with some energy left at the very end of the race, as a result when all $n-1$ riders die, we recalculate $v_{optimal}$ for the last rider remaining. This ensures that we use all the energy of the remaining player to its maximum potential.

In some cases our riders were able to make it to the finish line with some energy to spare. This happened in cases where there were many riders on a team. This configuration resulted in the distance where the $n-1$ player dies to be very close to the finish line. Thus taking a more conservative approach like taking the minimum energy in the players remaining when computing equation 3 will result in more than one rider finishing the race with a medal. This can be advantageous in tournaments where acquiring

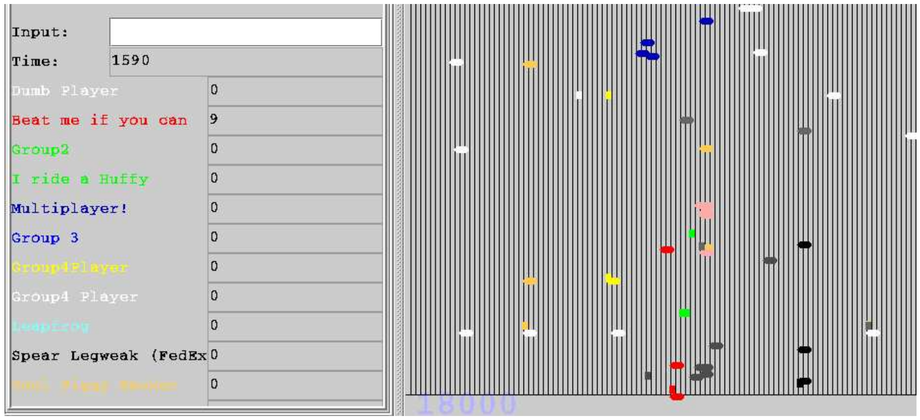

Figure 4: Many riders finishing

points is beneficial for the overall rank. Figure 4 demonstrates this situation.

3 Tournament Analysis

During mini-tournaments ran in class and self-made tournaments, we performed particularly well, winning most of the time. However, we noticed that in some cases our calculations were too accurate and our final rider was dying with his last energy unit right on the finish line (or just short of it due to problems with double floating point precision). This was causing our player to not win any medals. In order to rectify this problem we added an arbitrary factor of 2 meters to the D in equation 3 so that we finish our energy just beyond the finish line.

This solution although solved the problem of dying just on the finish line, resulted in very poor performance in the tournament. The calculation for the optimal speed is based on distance and is used in all stages of the strategy (beginning and end). Increasing this value even by a small amount altered the velocity that our riders would travel at. This small change in velocity throughout the race affected our player drastically and made the calculation less effective. The table below summarizes the results from the tournament.

There are a few key points to note from the tournament results. First when the number of teams is large (T=10), we do better when the number of lanes is large and particularly bad when the number of lanes in small. When the lanes are reduced by 10, our average rank is 10, however when the number of lanes is set to 2R and 2*T*R we see a considerable improvement in rank. This indicates that there are still some interference problems that are occurring preventing us from forming a line quickly enough if at all.

We also perform a bit better when the number of riders per team is greater than 4. We have a winning percentage of 11.5% vs. only 4% when racing with less than 4

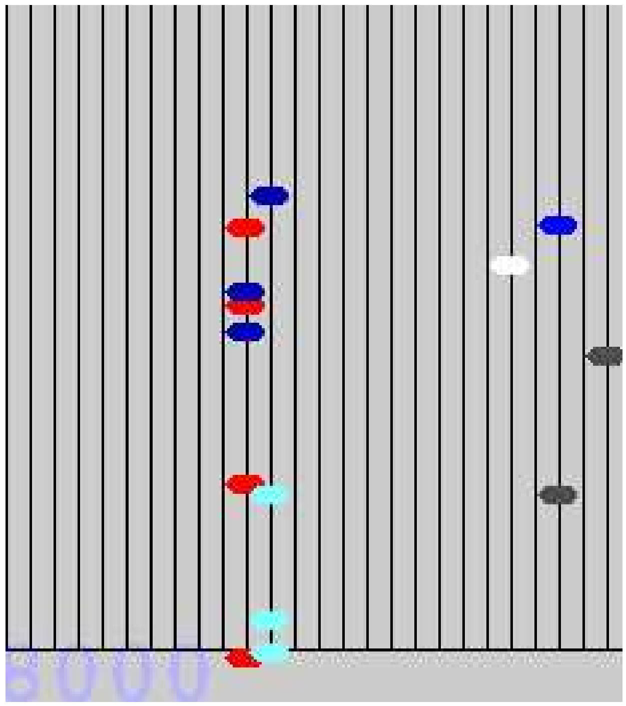

Figure 5: Rider finishes just on the finish line

Game	Avg. Rank	Wins	% Wins
T=10,L=2R	7.5	31	8%
T=10,L=R-10	10	6	12%
T=10,L=2*T*R	6	7	14%
R<5	8.25	8	4%
R≤5	6.75	23	11.5%
T=2	4	34	69%
T=4	4	37	43%

Table 1: Tournament results

riders. This is probably due to the fact that the increase in the number of riders also increases the number of lanes by a factor of 2. Additionally since most teams employ a "get into a line" strategy, there are more lanes available for movement. This allows us to move away from "blocking" players such that they do not interfere with us.

Finally, we perform really well when T=2 or T=4. When going head to head with another group, we win 69% of the time. We only come 2^{nd} to one other team *Group6PlayerM* which had a total of 35 wins, just one more win than us. Also, when T=4, we win 43% of the time, again coming only 2^{nd} to one team, *Group5K2*. We do particularly well when the number of teams is low due to the fact that destructive players have less advantage here. If we are going head to head with a destructive player, the destructive player will die out because of the extra energy it wastes on blocking us. We can then take advantage and win the race.

3.1 Areas of Improvement

From the tournament results it seems that are few improvements can be made. The initial change of adding a factor of 2 to D can be reduced. More importantly, most of the problems seem to occur in configurations when there is not enough space to move around and when "blocking" players can hinder our performance. The following changes are proposed that may improve our player and make it more robust:

- Calculating $v_{optimal}$ while considering varying energy levels of each rider instead of just taking the minimum. This can result in a more accurate computation.

- A Robust line forming strategy is needed so that we can always form line despite the interferences that occur during the beginning stages of the game.

- Avoiding blocking players is also needed so that players do not slow us down by coming in front of us, enabling others to win before us.

- Better lane changing in order to avoid being followed would also prove to be helpful in cases where we switch lanes simultaneously as with another team into the same lane. This causes disruption in the way the line of players are set up. A safer solution to maintain the integrity of the line would be helpful to avoid such detrimental cases.

- Taking advantage of drafting other players once we have formed a line during the beginning stages of the game could also be helpful since we can conserve some energy for our first rider and as a result may be able to go a bit faster during later stages of the game.

4 Member Contributions

We all worked together to develop the final player that was submitted.