Project 4: OLYMPIC ROAD RACE

The Adventures of Spear Legweak

Group 6

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1. Overview

Spear Legweak emerged from an obscure class project and after superhuman struggle against foes, humans and gods (a.k.a., simulator), won race after race, and finally reached legend status. This is his story.

Narrator: This project is designed on the lines of the Olympic road race. Given a team of riders, it is the responsibility of the team coach to make sure that his riders reach the finish line first, and gather as many medals as possible. The various parameters that the coach has to keep in mind are the number of teams participating in the race, the number of riders per team, the number of lanes available for the riders, the length of the race and the amount of initial energy provided to each rider.

Keeping the above variable parameters in mind, each team coach has to come up with a strategy based on the current state of the race and instruct his riders accordingly. In the case of a single player scenario where there is only one team participating, the race is more of a time trial where the coach tries to optimize the time taken by one of his riders to reach the finish line. On the other hand, in the multi player scenario, the coach has to ensure that his team of riders race to the finish line, while keeping account of the strategies of the other teams as well. Thus the multiplayer case is more conducive to interference from other teams, and coming up with a plan that is robust is as important as devising a strategy that aims at reaching the finish line first.

2. Ideas and Evolution

The key property that this race is based on is the fact that the rider who rides with no one else in front of him (within a distance of 5 meters), absorbs all of the wind resistance. A rider riding immediately behind another rider uses 30% less energy than a rider riding in front. The strategy that best utilizes this property to its advantage is one where all the riders of a team are lined up exactly one behind the other and ride in a single line all the way to the finish line. The rider in front dies after some amount of time when his energy runs out, and the rider right behind him now assumes the task of the leader. This was the first idea that was discussed in class, and we went on to implement this and make further enhancements that make it robust to the interference and other scheming strategies of the competing teams.

3. Players

3.1 Initial Single Player Strategy – The Spear Legweak Emergence

From the dark ages at the beginning of the project cycle, characterized by uncertainty and fear, an initial strategy came to be: this is briefly outlined below and consisted of all the riders traveling together in a single line; this way, they benefit from the drafting, and the energy savings can be utilized to travel at a higher constant velocity.

This strategy consisted of 2 stages:

a. Getting into line:
At the start of the race, the riders are placed in different lanes in a random manner. Thus the first big step is to get all your team riders to meet up in one lane and form a straight line where each rider (except for the leader) is behind the one ahead by exactly 2m. This makes sure that we get maximum energy savings of 30% while drafting.

Choosing the lane in which the riders are going to converge is the first decision to be made. We choose the midpoint of the locations of all the riders, and made that the meeting lane. The riders then move to the right or left and shifted lanes till they are in the meeting lane. The next step is to make the riders accelerate till they are positioned right behind one another. Thus this strategy does not allow the riders to accelerate or move ahead till they are in the meeting lane. If the rider's velocity is zero and he is at the starting line, then he is made to move ahead with maximum acceleration. If the rider's velocity is 1 (i.e. he is no longer on the start line, and has moved ahead a distance of 1m), he is no longer made to accelerate. If the rider's position is ahead of the length of the team when together (which is nothing but 2*num. of riders), then he is made to decelerate till the rest of the riders catch up. At the end of this, all the riders are aligned one behind the other and are ready to make use of the energy savings due to drafting.

b. Moving with optimal velocity:

Once the riders are aligned correctly, the next step is to compute the optimal velocity at which all the riders have to move in order for the last rider to reach the finish line as fast as possible. For the first deliverable, a speed of 9 was decided on. As is characteristic of such early ages, this was based on belief (trial and error) rather than scientific reasoning. The intuition that was emerging at that time was that riders ought to move with constant velocity during each stage. It was not clear whether the velocities should change from stage to stage, but that was the easiest thing to implement at that time.

Each rider then moves at this velocity, till his energy runs out. Thus all but the last rider die at different stages of the game, and the last rider races to the finish line. The rider to finish the race became known as Spear Legweak. It was only the beginning…

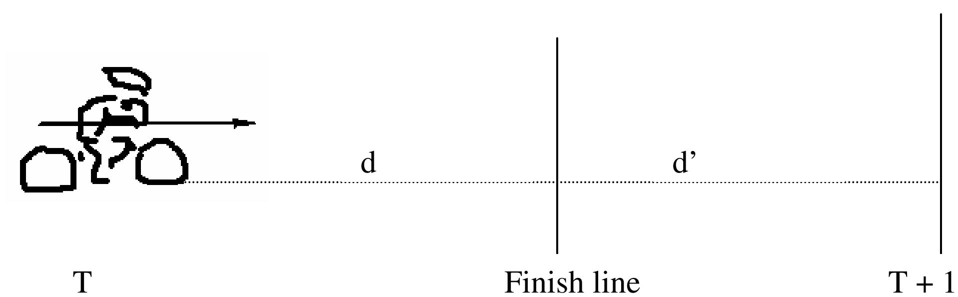
3.2 Optimal Single Player Strategy – The Spear Legweak Identity

Spear Legweak, although animated by high ideals and filled with enthusiasm, failed to perform in the first couple of classes, mainly because of simulator glitches, which left him and his team very frustrated. However, Spear Legweak was determined to win. He hired a capable team of engineers (group 6) to fine-tune his strategy.

Already as early as the second discussion class, the team's engineers had gone through a lot of math, which not only impressed Spear Legweak (who is more of a humanities person) and gave him confidence he made the right decision when hiring his engineers, but also opened up the era of optimal strategies in the history of road races. Group 6 found out that the speed of the column ought to be constant throughout the race for optimal finishing time. Their work has been recorded in section 1 of the appendix. The speed for a team of R riders is $v(R) = [(10/7)(E/D)(1 - 0.3)^R]^{2/3}$. Confident of the correctness of their calculations, group 6 shared the formula with the other teams, to level the playing field. Many other coaches soon adopted the optimal speed strategy themselves.

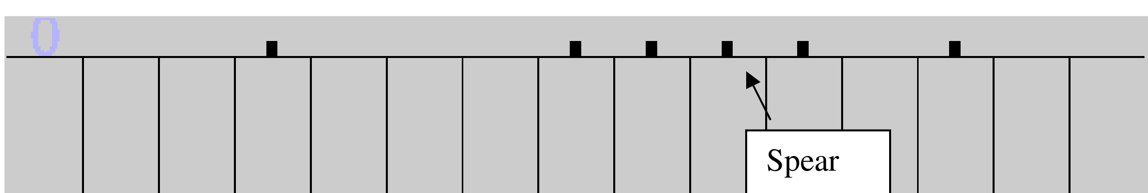
Still, the engineering team of Spear Legweak was not content. They realized that their derivation was under a simplified model. In reality, all the players in the column were not at exactly the same position (in fact, the first and last were separated by R*2m). Not all of them had exactly the same energy. There were interactions with other teams beyond the control of Spear's coach, such as forced slowing down by other riders ahead. And there was the column formation, which the mathematical model had completely ignored. This latter issue was the most worrying for the engineers of group 6, who realized that a good strategy might make the difference between the yellow jersey and the shroud of death at the finish line. This problem was exacerbated by the fact that many teams were now using the same formula, and Spear Legweak needed an edge over his opponents.

The first modification made by group 6 was to squeeze the last drop of advantage out of the optimal speed: the computation became focused on Spear Legweak. He was the last rider in the column, and he was the one supposed to finish the race. Everyone else was there only to help him. All computations were done using Spear's energy and distance to the target, and the optimal speed was readjusted at each step of the simulator, to smooth out any inaccuracies caused by the deviations from the ideal mathematical model. With this optimization, Spear always set the pace of the race, even though he was last in the column. The strategy worked remarkably well. True, Spear died one meter from the finishing line when the race was run in class, but he was ahead, and the engineers were satisfied: Spear and his team had made optimal use of their energies. However, Spear was not very happy with dying a bike's length away from finish line, and under his pressure, the engineers devised a simple fix: they told Spear to ride to a virtual finishing line that was actually a few meters beyond the real finishing line. Spear would run out of energy before the virtual finishing line, and not even realize that he already crossed the real finishing line. As long as he won, this little deceit didn't bother him very much. The engineers figured out that speeds were computed correctly, but the reason that Spear died right before the finish line was that his energy would last him *only* up to the finish line. Since the simulator's step did not have infinitesimal granularity, it actually required the riders to have energy for an entire step. While Spear had enough energy to cross the finishing line, he did not have enough to perform an entire step:

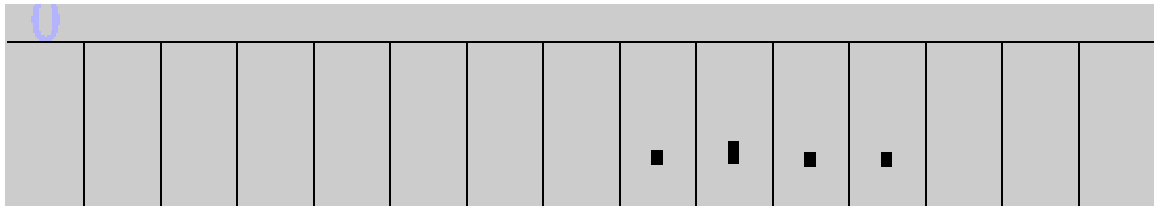


For example, in the figure above, at simulator time T, Spear has enough energy to go a distance d, but not enough to go a distance d + d', which is where he would be at time step T+1 of the simulator. Hence, all that's left of him by time T+1 is a corpse. The engineers then set the virtual finish line to be beyond this troublesome d'. Worst case, their reasoning goes, Spear would be right before the finish line (d close to 0), and he would need to last out another simulator step (where d' approaches the distance he would travel in a time unit). Hence, they placed the virtual finish line to be a distance v beyond the finish line, where v is Spear's current speed. This worked as expected, as Spear never ever died again. And while he would need to sacrifice some speed along the race to save up extra energy for this last leap, the energy that he finished with was small (about $v^{2.5}$ / 2 on average), roughly couple hundred on the standard race track (where the total energy was 5 million). The engineers realized that Spear would get beaten by teams which took the risk and didn't save up any energy for the finish, but they figured it's better for Spear to safely finish the race and get *some* medal. Which he did. And the teams who did not take such precautions sometimes died before finishing, so Spear would sweep gold. And he would also win gold even when all the other teams finished as well, because of the other tunings in his strategy.

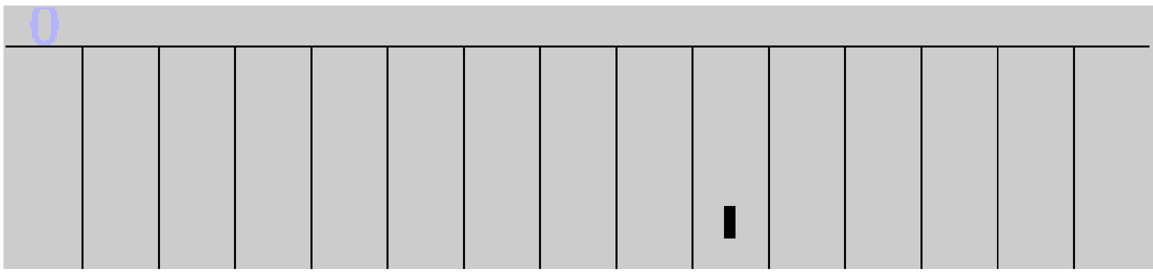
The engineering team was ready for their second challenge: the starting phase. Who *is* Spear Legweak? The gods place the team's riders randomly at the starting line. How should the column be formed? Who should be Spear Legweak? The engineers turned again to math for help. But math was on sabbatical. So they did the next best thing: they approximated. They took a simplified model, and they tried to understand what to do at the starting line: should everybody come together on the starting line as they did up to now? This is clearly what would be most energy efficient, since at least one rider must at any time face the wind, but all other riders would be shielded for as long as possible. But Spear wanted to win. He is hot-headed, and wants to start biking from the very start, to save up time. The column may very well form as we go, said Spear. The engineers analyzed the model in section 2 of the appendix, and saw that Spear was almost right: in a two rider scenario, the riders ought to meet at the midpoint lane, ride out with v(1) until they meet, and then ride out with v(2) till the end of the race. The engineers would of course not admit to Spear that he had the right intuition, and instead came up with a fancy strategy that combined Spear's desire to ride out fast, with the need to be conservative about wasting energy. They also wanted a robust way for the riders to form the column, which would not be prone to interference from competitors. Since 2 players should meet at the midpoint, they generalized this to the median of R players. The well-known "post office placement" problem is the following: given n cities on a line and their coordinates, where should a post office be placed so that the sum of the distances from the post office to each city is minimized? The answer turns out to be the median city. Hence, the meeting lane for the riders was chosen to be that of the median player. The median player then was recognized as Spear Legweak, and would be perpetually at the tail of the column. The other riders would always add themselves to the head of the column (like a stack). The strategy was as follows: each rider tries to move towards the lane where the column is forming. They go at speed v(R). Once they reach the lane, they are ahead of the column, and they switch to speed v(1) < v(R). The column does the opposite: they go at speed v(1), until they notice a team rider that is ahead. Then the column switches to speed v(R) and catches up to the rider, who, recall, now rides at speed v(1). The reason that nobody stops is to avoid slowing the column down, which, according to the intuition gained from section 2 of the appendix, ought to go at a speed on the order of v(1) or v(R). So, when the column catches up with the rider ahead with speed v(1), the entire column automatically continue with speed v(1). Once all the riders are in the column, the column maintains the optimal speed v(R) throughout the race (where v(R) is recomputed continuously according to Spear's energy and position, as mentioned earlier).



Time 0: Initial placement on the board.



Time 19: Column already contains three riders. Lance is at the end of the column. The column goes slower than the ‘free’ riders, so these can get in front of it.



Time 26: The column is fully formed.

This get-together strategy converges very fast, even though all riders ride at or close to the optimal speed. We are usually among the first teams to have a column fully formed, which of course pleases Spear’s teammates, who don’t ride in the wind for too long. Spear himself is shielded almost from the start, since the first rider to join the column will ride in front of him. In the example, above, this happens at round 13.

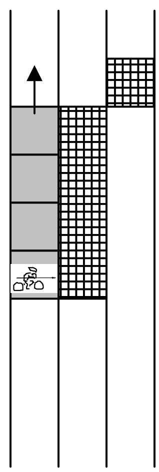
Spear is thus leading the column only for a few rounds at the beginning (when the column is just himself) and on the last stage, when everybody else is dead (the length of this last stage is exponentially small in the number of riders). The choice to keep our ‘most likely to succeed’ graduate (that is, Spear Legweak) at the end of the column is motivated by the discussion in section 3 in the appendix. The rider with most energy is best off at the end, and the one with least energy at the front.

3.3 The Seclusion Multiple Player Strategy – The Spear Legweak Supremacy

Thus prepared, Spear takes on the world. And the world is full of evil. There are teams that block, there are teams that trail, and there are teams that simply get in the way because they don’t know any better. Spear has invested a lot of his [engineering team’s] intellectual effort into a good strategy for his team, and he does not want to see it disturbed by such critters. He is in fact so confident, that he does not need to trail anyone for an extra energy boost. He just wants to be left alone. The engineers get back to work.

They first make sure that the column is formed properly. This is key to the success of the strategy. Since many other players will have speeds similar to the ones used by Spear’s team, it may happen that a foreign rider, riding at the same speed alongside one of our ‘free’ riders, inadvertently blocks the free rider to move lanes and join the column. The solution is for the free riders to detect such situations and accelerate and overtake the troublesome parallel rider. He should not decelerate, since otherwise, he may end up behind the column once he reaches Spear's lane, instead of ahead.

Spear's team also tries to avoid being blocked, since it must use the optimal speed at all times, and not be forced to ride behind a slower player. Also, Spear is not willing to help out any of the competitors: he wants nobody to trail him for wind shielding. These situations are detected, and if possible, the entire column switches lanes simultaneously. 'Possible' above means that the column would not break up when switching lanes. The conditions that are sufficient for a 'safe' column switch are illustrated in the figure below:



If Spear's column (shaded region) wants to move a lane to the right, then the checkered region in the figure needs to be unoccupied. While the reasons for the region in the lane immediately to the right are obvious, we also need to make sure the little area on the second lane to the right is free. This little area extends from where our column leader is, to one bike length ahead. If there was a player there, he could switch to the left at the same time we switch right, and block the column leader from changing lanes. On the other hand, if the opponent rider is behind this little area on the second lane to the right, one of the riders in Spear's column will be ahead of him and thus win the tie and occupy the middle lane. The move is thus safe. A symmetric situation occurs to the left.
If it has a choice and needs to move, Spear's team always moves to the left, since that region of the track is usually the freest and future interference is unlikely. The engineers did not bother moving the column randomly even if it was possible, since that seems to serve no purpose and just makes following the race frustrating for the eyes.

4. Other approaches and reproaches

4.1 No Player Strategy – The [avoided] Spear Legweak Demise

There were many global visions, national strategies, local techniques, and back-alley tricks proposed during the class discussion sessions. Spear's team carefully evaluated many of them, but ended up using very few ideas. A large part of the discussion focused on parasitic strategies, which were meant to take advantage of other teams in dodging wind resistance. Group 6 found few practical incentives to use such approaches, and even less theoretical reasons: the best wind resistance comes from your own riders, which do not try to move out of your way constantly, as happens when you are trying to follow a competitor. So, unless a team forms a column and rides behind another team's player, a rider of the team is always better off following one of his team's own riders. The idea to form a column and ride behind some other team seems appealing, but the engineers were reluctant to do anything that could tampered with the optimal speed strategy: they found that slight changes in the speed decreased the performance of the team dramatically (from winning gold to coming in at the middle of the platoon).

For similar reasons, they dismissed the 'evil' blocking strategies. Unless part of a two-team tournament, there are no game-theoretic incentives to sacrifice one's own team performance to harm another team. Furthermore, disruption strategies are very complex to implement and easy to avoid by opponents.

4.2 Future Player Strategy – The Spear Legweak Retirement

The engineering team was overworked and underpaid. If they had been treated better, they would have loved to add extra enhancements to the legacy of Spear Legweak.
They would probably overhaul the starting strategy, which, in terms of optimization, trails the fine-tuning of the long-term strategy. While they are content with the way the column is formed as a first-order approximation, a better analysis with more riders would be in order. As noted in appendix section 2, two riders ride alone with speed v(1) and then continue with speed v(2) after meeting. As a second-order improvement, this should be generalized to a strategy where riders form smaller columns, and smaller columns eventually converge into the large column. Each intermediate small column with r riders will ride with speed v(r).

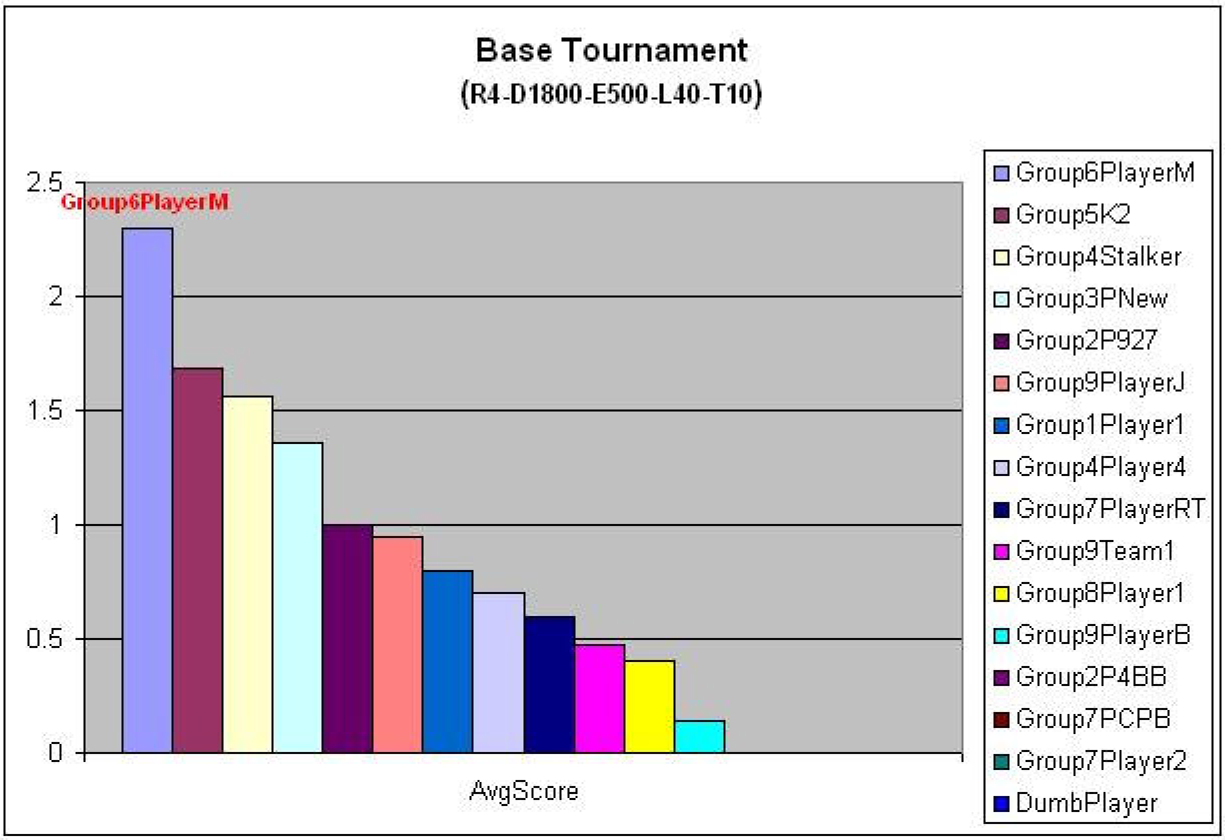
Another direction to explore is taking advantage of other riders by forming a column and then trailing other riders. While this was discussed above and not implemented in the current player, a careful analysis of the trade-off between sacrificing optimality of the speed and gaining energy from trailing could yield a player that is absolutely superior to both the optimal strategy teams, and the stalker teams.

Finally, a question that the engineers pursued was how (if at all) the optimal strategy needs to be altered as the number of riders per team increases. The discussion of section 4 in the appendix is enlightening in that regard. It turns out that for more than 8-9 riders, the team gains almost no benefit from a longer column. Extra players are better off implementing a separate or related game alongside the main column, such as protecting the column in the early column forming phase, or sheltering the column from blocking, and trailing in later stages. Or if there are many extra players, they can form a second column and thus try their chance at a medal.

5. Tournament Analysis

The engineers are very pleased with the performance of Spear Legweak. The team has proved itself in a variety of scenarios and has come out in flying colors. We will analyze the team's run in some of the interesting tournaments below:

5.1 Base tournament



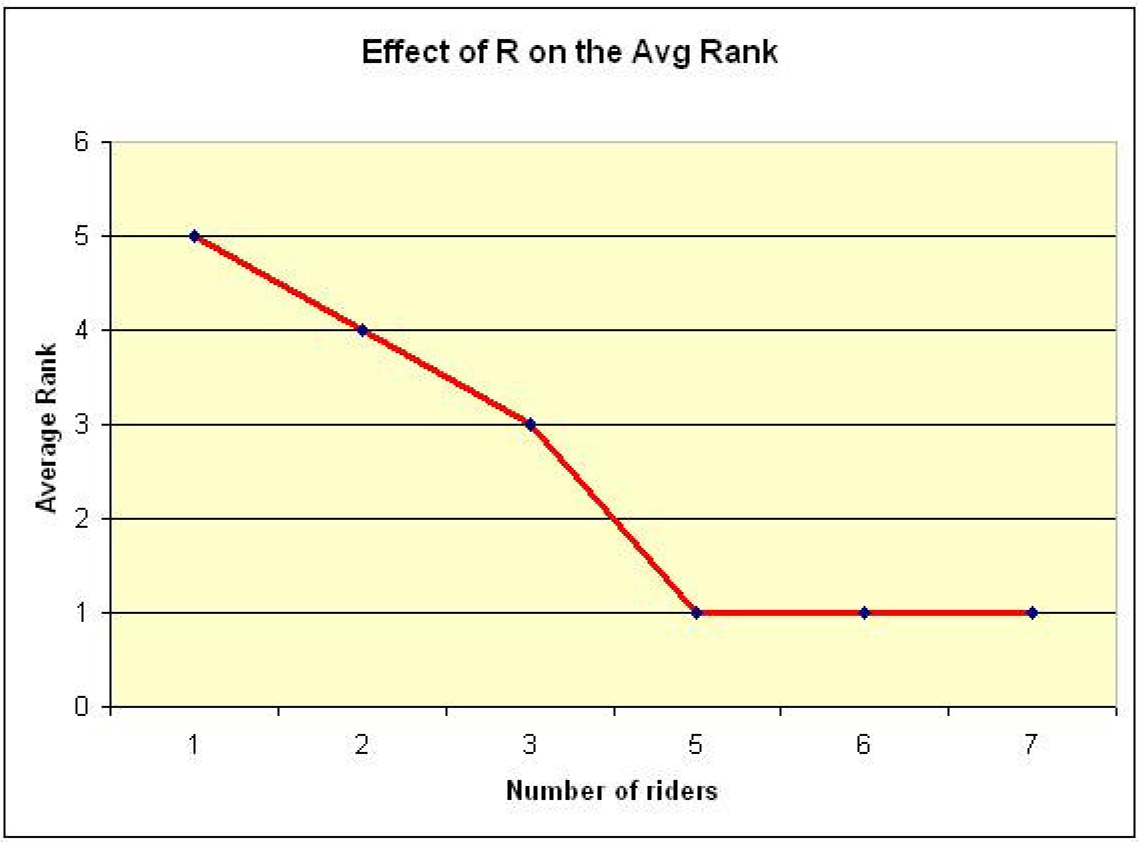
We see that in the base tournament where the parameters were similar to the standard one that was used in class (E and D were scaled down, however the E/D ratio is the same), Spear Legweak ***reigned supreme*** with the highest average score. This was expected since the riders were using the optimal strategy which was very finely tuned.

5.2 Effect of the number of riders (R)

An interesting pattern that the engineers noticed is the performance of the team varies with the number of riders per team. As depicted in the graph below, Spear Legweak ranked fifth, fourth and third when the number of riders were fewer, i.e 1, 2 and 3 respectively. The team excels and ranks first with the increase in the number of players namely 5, 6 and 7.

This behavior can be understood by looking at the math provided by the engineers as an appendix. The scenarios with fewer riders are beneficial to strategies such as trailing and less advantageous to the riders-in-a-single-line strategy. The energy savings is not significant enough to make the optimal team speed aggressive enough. There is a significant change in the optimal velocity with the increase in riders and Spear's team consistently reaches the finish line faster than its competitors. This explains the team winning the gold medal time and again.

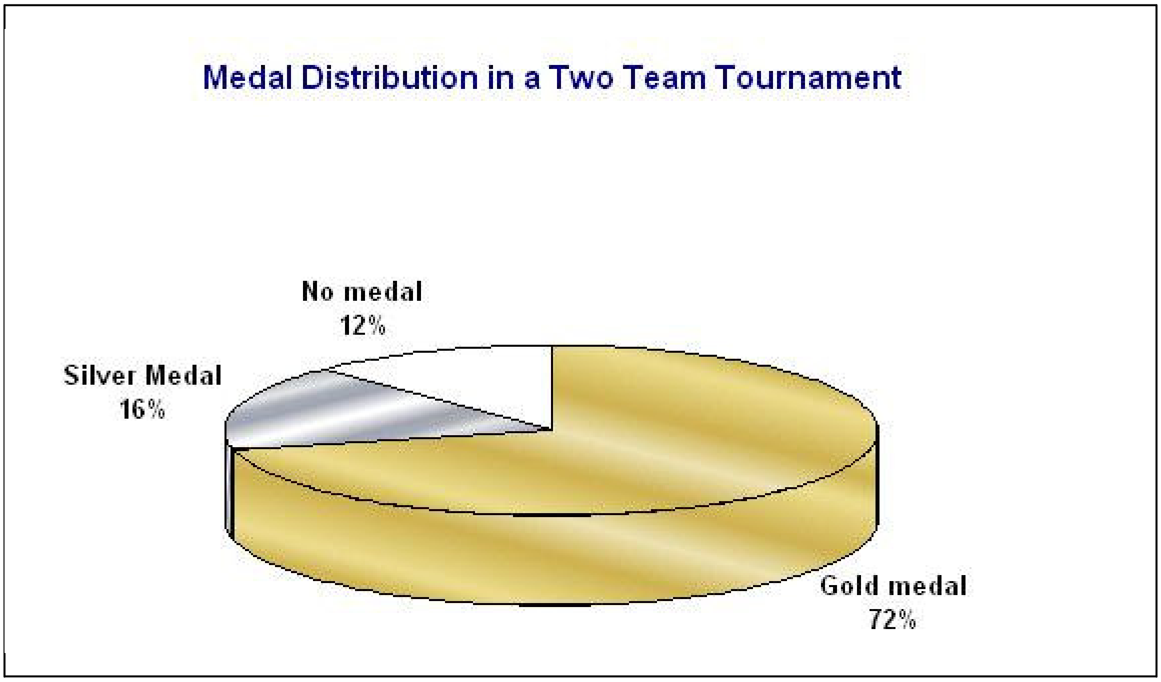
However we see that when the number of riders is 9, the team's performance falls again. This is because there is an upper limit to the increase in optimal speed with the increase in number of riders. As R goes above 7 or 8, the increase in the optimal speed is marginal. Additionally, the overhead of getting all the riders into the same lane and in line increases and we see an overall decrease in the team's performance.



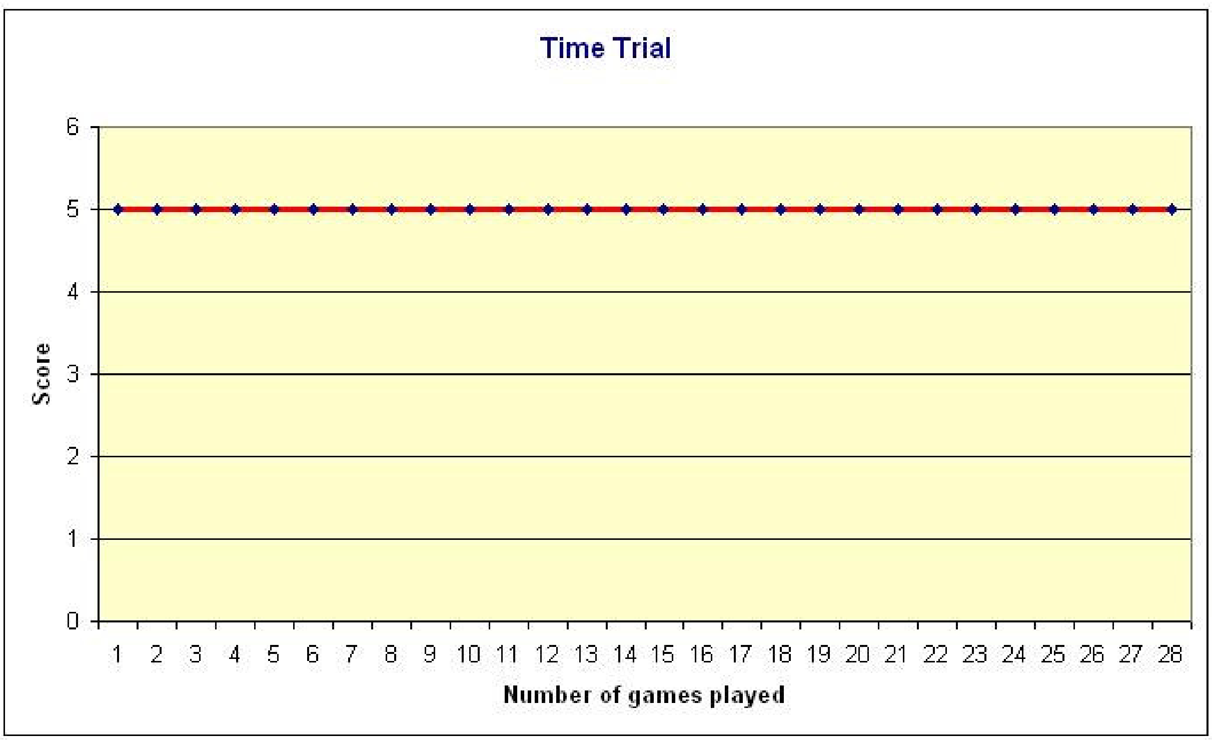
5.3 Performance in the 2-Team Tournaments

The 2-team tournament results were very satisfactory. In the 49 games played, Spear's team won the gold medal 72% of the time, the silver 16% of the time, and unfortunately did not win any for about 12% of the time.

It is also very heartening to see that in the times we did not win any medal, we were competing with the best of the competition, which also followed the same optimal strategy.



5.4 Single Player Tournaments – Time Trials



The goal in the single team tournaments is not about securing as many medals as possible but about making your rider cross the finish line keeping in consideration minimizing the time taken to do so. The tournament was run 28 times for Spear's team and in all the runs, Spear raced to the finish line in the least amount of time.

The tournament results do not provide us with the time taken for the riders to complete the race, and is instead ranked based on the average score (the number of medals won by each team). This is not of much use as this scenario does not test the medals won, and instead tests the efficient utilization of time. Additionally, the teams that got a higher score due to many of their riders crossing the finish line have to be using the sub-optimal strategy. The graph below shows the performance of our team, where all but Spear die before the completion of the race, but Spear races to the end without fail and gets the gold medal worth 5 points.

5.5 The immortality of Spear Legweak

The final feature of Spear Legweak that we want to bring to your notice is that he never dies. Irrespective of the variable parameters or the interference and destructive tendencies of its competitors, Spear's team always survives!

6. Individual Contributions

All three of us contributed equally to the development of Spear's team. The engineers put their heads together at each meeting and made Spear the legend that he is. Additionally, Bogdan and Sowmya wrote up the report, and again the engineers came collectively for the presentation of their star player.