

Lecture 1: January 19, 2017

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1 Introduction

This is a template file for scribe notes for COMS 6998. This class is about Boolean functions $f : \{0, 1\}^n \rightarrow \{0, 1\}$. Here is some filler text.

Check out the style files `6998.sty` and `6998-thm.sty` (in the same directory where you found this file); those files contain macros, etc that can make your notes easier to write and more attractive to read.

Here's an example of an embedded link (in this case, a link to a free online version of Wegener's "blue book"): *[The Complexity of Boolean Functions](#)*.

2 Some hints about organization

It's a good idea to organize your scribe notes into sections according to logical break-points in the material. Similarly, it's a good idea to organize technical material into lemmas, claims, theorems, et cetera. LaTeX is great at automatically handling references to material in your text. Here's a simple example:

Theorem 1. *This is a dummy theorem.*

By labeling the theorem in the LaTeX source it's easy to refer to it later in an automatic way (for example, this parenthetical references Theorem 1 — see the LaTeX source).

3 Some hints about exposition

You should strive for clarity in your writeup of scribe notes — don't be too concise, but also avoid unnecessary over-explanation. A good imaginary reader to keep in mind is a friend who's smart and mathematically mature, but doesn't have any special expertise or knowledge of this area.

4 Some hints about presentation

Pictures/diagrams/figures are extremely helpful! It's a good idea to include as many of these as you need. You can draw them in LaTeX itself or import files, whatever you prefer — the key thing is that your figures are legible and informative. See Appendix A for an example of a figure. (It's taken out of context so don't worry about the content — but hopefully you are convinced that in whatever context it came from, the figure helps explain what is going on!)

Everybody likes to read good-looking mathematics; or rather, nobody likes to read math that's ugly or hard to parse. (Subjectivity alert) Here is an example of some reasonably good-looking math:

$$(16n)^s = 2^{\log(16n) \cdot 2^n / 2 \log(n)} = 2^{\frac{4 + \log(n)}{2 \log(n)} \cdot 2^n} < 2^{0.51 \cdot 2^n} \ll \left(1 - \frac{1}{2^n}\right) \cdot 2^{2^n}.$$

Finally, a typical set of scribe notes should provide references to relevant papers, books, etc. as appropriate. This sentence contains several references to various papers [Sha49, Sub61, Hås86].

References

- [Hås86] Johan Håstad. Almost optimal lower bounds for small depth circuits. In *Proc. 18th Annual ACM Symposium on Theory of Computing (STOC)*, pages 6–20. ACM Press, 1986. 4
- [Sha49] Claude E. Shannon. The synthesis of two-terminal switching circuits. *The Bell System Technical Journal*, 28:59–98, 1949. 4
- [Sub61] B. A. Subbotovskaya. Realizations of linear functions by formulas using $+$, $*$, $-$. *Soviet Mathematics Doklady*, 2:110–112, 1961. 4

A An example figure

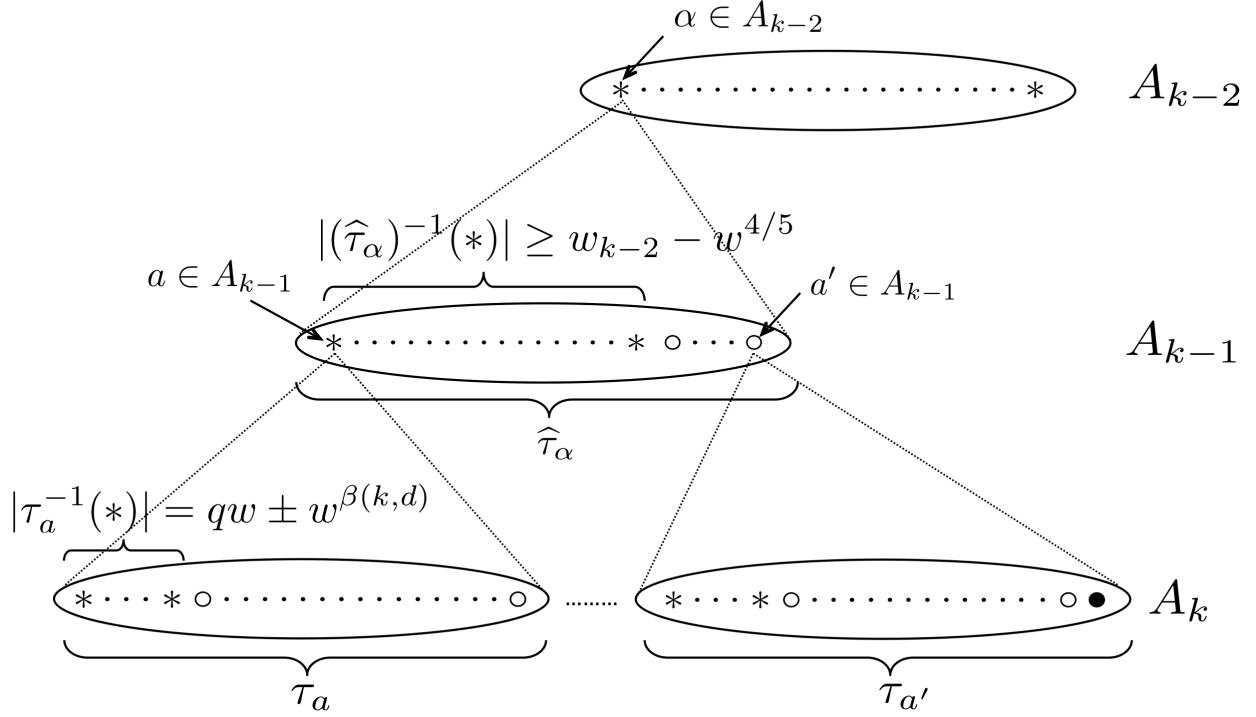


Figure 1: The figure illustrates a typical $\tau \in \{\bullet, \circ, \ast\}^{A_k}$. For $a \in A_{k-1}$, τ_a is a block of length w_{k-1} , i.e. a string in $\{\bullet, \circ, \ast\}^{w_{k-1}}$. We may think of the block τ_a as being located at level k . By Condition (1) of Definition 72, for every $a \in A_{k-1}$ we have that $|\tau_a^{-1}(\ast)|$, the number of \ast 's in τ_a , is roughly $qw = \tilde{\Theta}(\sqrt{w})$. The lift $\hat{\tau}$ of τ is a string in $\{\bullet, \circ, \ast\}^{A_{k-1}}$, and for $\alpha \in A_{k-2}$, $\hat{\tau}_\alpha$ is a block of length w_{k-2} . We may think of the block $\hat{\tau}_\alpha$ as being located at level $k-1$. As stipulated by equation (86), for every $\alpha \in A_{k-2}$, the string $\hat{\tau}_\alpha$ belongs to $\{\ast, \circ\}^{w_{k-2}}$. By Condition (2) of Definition 72, for every $\alpha \in A_{k-2}$, we have that $|(\hat{\tau}_\alpha)^{-1}(\ast)|$, the number of \ast 's in $\hat{\tau}_\alpha$, is at least $w_{k-2} - w^{4/5} = w_{k-2}(1 - o(1))$. Finally, we observe that equation (55) and Condition (2) of Definition 72 imply that $\hat{\tau}_\alpha = \ast$ for every $\alpha \in A_{k-2}$.