

## Example of FOL Theorem Proving

Axioms:

- F<sub>1</sub> =  $x = x$
- F<sub>2</sub> =  $x = y \rightarrow y = x$
- F<sub>3</sub> =  $x = y \wedge y = z \rightarrow x = z$
- F<sub>4</sub> =  $(x + y) = (y + x)$
- F<sub>5</sub> =  $(x + (y + z)) = ((x + y) + z)$
- F<sub>6</sub> =  $(x + 1) = y \wedge (y + 1) = z \rightarrow z = (x + (1 + 1))$
- F<sub>7</sub> =  $x + 0 = x$
- F<sub>8</sub> =  $x + 1 = (x + 1) + 0$

Number n = ( 1 + (1 + ... 0) ; number n is represented as n successive  
; additions of 1 to 0

Prove  $G = 2+2=4$  is a logical consequence of the axioms!

Clauses:

- (1): ( = (x, x) )
- (2): ( ~(x, y) = (y, x) )
- (3): ( ~(x, y) ~(y, z) = (x, z) )
- (4): ( = ( +(x, y), +(y, x) ) )
- (5): ( = ( +(x, +(y, z)), +(+(x, y), z) ) )
- (6): ( ~(+(x, 1), y) ~(+(y, 1), z) = (z, +(x, +(1, 1))) )
- (7): ( = ( +(x, 0), x ) )
- (8): ( = ( +(x, 1), +(+(x, 1), 0) ) )

~G(9): ( ~( +( +(1, +(1, 0)), +(1, +(1, 0))), +(1, +(1, +(1, +(1, 0)))) ) )  
; |<--2-->| |<--2-->| |<----4---->|

Ok, now run your theorem prover.