Choosing Determinacy: Combining Concurrency and Timing in the Sparse Synchronous Model

Stephen A. Edwards

Boolean Functions as a Table

	\mathbf{B}	Ĺ	1 5		/ /		5				
W	Х	Y	Ζ	а	b	с	d	е	f	g	
0	0	0	0	1	1	1	1	1	1	0	
0	0	0	1	0	1	1	0	0	0	0	
0	0	1	0	1	1	0	1	1	0	1	
0	0	1	1	1	1	1	1	0	0	1	
0	1	0	0	0	1	1	0	0	1	1	
0	1	0	1	1	0	1	1	0	1	1	
0	1	1	0	1	0	1	1	1	1	1	
0	1	1	1	1	1	1	0	0	0	0	
1	0	0	0	1	1	1	1	1	1	1	
1	0	0	1	1	1	1	0	0	1	1	
1	0	1	0	0	0	0	0	0	0	0	
1	0	1	1	0	0	0	0	0	0	0	
1	1	0	0	0	0	0	0	0	0	0	
1	1	0	1	0	0	0	0	0	0	0	
1	1	1	0	0	0	0	0	0	0	0	
1	1	1	1	0	0	0	0	0	0	0	

For *n* Boolean inputs and *m* Boolean outputs, Fach of 2^n rows lists the *m*

Boolean outputs for that row's input combination

Each possible input combination appears in exactly one row

It is a total function: $2^n \rightarrow 2^m$

Acyclic Networks of NAND2 Gates



https://www.electronics-tutorials.ws/logic/universal-gates.html

Directed Acyclic Graph of Two-input NAND gates

Primary inputs: no incoming edges

All others: two incoming edges

Semantics: set value of each primary input; in topological order, set each node's value to the NAND of the values of its two incoming edges

Can compute any Boolean function

Deterministic: Assignment of each node's value depends only on the primary inputs, not the particular topological order chosen

Deterministic Finite Automaton as a Table



After Hopcroft and Ullman, Introduction to Automata Theory,

Languages, and Computation, 1979

- List of states, some are accepting
- A start state
- List of inputs
- Complete table of transitions (state, input) → state

Deterministic if, for each state and input, there's exactly one next state

Synchronous Digital Logic

DAG with three types of nodes:

- NAND2: two incoming edges
- flip-flop: one incoming edge
- primary input: no incoming edges

Every cycle in the graph must pass through a flip-flop

In each cycle, primary input nodes set to new value, flip-flop nodes set to input in last cycle (false in first)

NAND2 nodes evaluated in topological order, ignoring flop-flop input edges



Turing Machine





- A tape of symbols
- A head that can read and write symbols and move left or right
- A state register

• A table of instructions: (state, symbol) \rightarrow (state, symbol, left/right) Deterministic because there's exactly one thing to do at each step

expr ::= expr expr | λ variable . expr | constant | variable | (expr)



Kozen, Church-Rosser Made Easy, Fundamenta Informaticae, 103(1–4), 2010

two = $\lambda f \cdot \lambda x \cdot f(f x)$ three = $\lambda f \cdot \lambda x \cdot f(f(f x))$ five = $\lambda f \cdot \lambda x \cdot f(f(f(f(x))))$ plus = $\lambda m \cdot \lambda n \cdot \lambda f \cdot \lambda x \cdot m f(n(f x)))$

plus three two

Expand plus

expr ::= expr expr | λ variable . expr | constant | variable | (expr)



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plus three two

 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

 β -reduce (λ m ...) three

expr ::= expr expr | λ variable . expr | constant | variable | (expr)



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plus three two

 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

 $(\lambda n.\lambda f.\lambda x. \text{ three } f(n f x)) \text{ two}$

 β -reduce (λ n ...) two (could have expanded three)

expr ::= expr expr | λ variable . expr | constant | variable | (expr)



Kozen, Church-Rosser Made Easy, Fundamenta Informaticae, 103(1–4), 2010

two = $\lambda f \cdot \lambda x \cdot f(f x)$ three = $\lambda f \cdot \lambda x \cdot f(f(f x))$ five = $\lambda f \cdot \lambda x \cdot f(f(f(f(x))))$ plus = $\lambda m \cdot \lambda n \cdot \lambda f \cdot \lambda x \cdot m f(n(f x)))$

plus three two

 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

 $(\lambda n.\lambda f.\lambda x. \text{ three } f(n f x)) \text{ two}$

 $\lambda f.\lambda x.$ three f (two f x)

Expand three and beta reduce twice (could have expanded two)

expr ::= expr expr | λ variable . expr | constant | variable | (expr)



Kozen, Church-Rosser Made Easy, Fundamenta Informaticae, 103(1–4), 2010

two = $\lambda f \cdot \lambda x \cdot f(f x)$ three = $\lambda f \cdot \lambda x \cdot f(f(f x))$ five = $\lambda f \cdot \lambda x \cdot f(f(f(f(x))))$ plus = $\lambda m \cdot \lambda n \cdot \lambda f \cdot \lambda x \cdot m f(n(f x)))$

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 $(\lambda n.\lambda f.\lambda x. \text{ three } f (n f x)) \text{ two}$

 $\lambda f.\lambda x.$ three f (two f x)

 $\lambda f.\lambda x. f(f(f(\underline{two} f x)))$

Expand two and beta reduce twice

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 $\lambda f.\lambda x. f(f(f(\underline{two} f x)))$

 $\lambda f. \lambda x. f(f(f(f(x)))))$

Normal form (nothing more to do)

expr ::= expr expr | λ variable . expr | constant | variable | (expr)



Kozen, Church-Rosser Made Easy, Fundamenta Informaticae, 103(1–4), 2010

two = $\lambda f \cdot \lambda x \cdot f(f x)$ three = $\lambda f \cdot \lambda x \cdot f(f(f x))$ five = $\lambda f \cdot \lambda x \cdot f(f(f(f(x))))$ plus = $\lambda m \cdot \lambda n \cdot \lambda f \cdot \lambda x \cdot m f(n(f x)))$

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 $\lambda f. \lambda x. f(f(f(f(x)))))$

five

This is "five"

Many reducible sub-expressions: Church-Rosser: all choices OK

$$\left(\left(\lambda x \cdot \left((\lambda w \cdot \lambda z \cdot + w z) 1\right)\right) \left((\lambda x \cdot x x) (\lambda x \cdot x x)\right)\right) \left((\lambda y \cdot + y 1) (+ 2 3)\right)$$

$$\lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda y \quad \lambda x \quad \lambda x \quad \lambda y \quad \lambda y$$

Many reducible sub-expressions: Church-Rosser: all choices OK



Kahn Process Networks

 $\begin{array}{c}
T^{2} & h \\
T^{2} & f \\
T^{2} & f \\
T^{2} & f \\
T^{2} & h \\
T^{2} & h$

Network of concurrent processes communicate through FIFOs

Blocking reads; non-blocking writes

Sequence of data values passed through each FIFO is deterministic

```
process f(in int u, in int v,
          out int w) {
  int i; bool b = true;
                                process h(in int u, out int v,
 for (::) {
                                           int init) {
    i = b? wait(u) : wait(w);
                                   int i;
    printf("\%i\n", i):
                                   send(v, init);
   send(i, w);
                                   for (::) {
    b = !b;
                                     i = wait(u);
                                     send(i, v);
process g(in int u, out int v,
          out int w) {
  int i; bool b = true;
                                channel int X, Y, Z, T1, T2;
 for (;;) {
    i = wait(u);
                                f(Y, Z, X);
    if (b) send(i, v);
                                g(X, T1, T2);
    else
          send(i, w);
                                h(T1, Y, 0);
                                h(T2, Z, 1);
    b = !b:
```

Discrete-Event Simulation: Verilog



Discrete-Event Simulation: Verilog

- 1. Select, remove, and execute earliest pending event *e* from queue
- 2. At an event @(), mark successor as sensitive
- On assignment v =, schedule all events sensitive to the variable
- 4. On delay #, schedule successor in the future



#7

Nondeterminism in Verilog

```
      module race;
      10 first

      reg a;
      10 second

      initial begin #10; a = 1;
      20 first

      #10; a = 0;
      30 first

      #10; a = 1; end
      30 second
```

always @(a) \$display("%0t first", \$time);

```
always @(a) $display("%0t second", $time);
```

endmodule





Adam Kepecs, Cold Spring Harbor Laboratory [Lak et al., Neuron 84(1), 2014]

Bpod: An Open Hardware Platform for Behavioral Monitoring and Control



Sanworks.io, spun out of Kepecs' lab. Teensy 3.6: ARM Cortex M4, 180 MHz

SSM: The Idea



```
training gate valve led =
  let timeout = new 0
  valve <- 1
  delay (ms 100)
  valve <- 0
  after (s 10), timeout <- 1
  wait gate || timeout
  if updated timeout
    failed <- failed + 1
  else
    led <- 1
    after (ms 100), led <- 0
   wait led
```

SSM: Wishlist

Deterministic formal semantics

Explicit model-time delays only; platform-independent timing above some minimum delay (synchronous logic)



"Bare metal" microcontroller implementations: hardware counter/timer drives timing, timer interrupts for scheduling



Concurrency

Time in seconds Can add, subtract, multiply, and divide time intervals



Time is quantized; quantum not user-visible Quantum might be 1 MHz, 16 MHz, etc. Integer timestamps thwart Zeno



Time is quantized; quantum not user-visible

Program thinks processor is infinitely fast: execution a sequence of zero-time instants (hence "synchronous")

Every instruction that runs in an instant sees the same timestamp



Time is quantized; quantum not user-visible

Program thinks processor is infinitely fast: execution a sequence of zero-time instants (hence "synchronous")

Nothing happens in most instants (hence "sparse")



```
blink led = led is mutable; can be scheduled

loop

after ms 50,

led <- not (deref led)

wait led
```



```
blink led = led is mutable; can be scheduled

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after ms 50,

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wait led
```



```
blink led =

loop

after ms 50,

led <- not (deref led)

wait led
```

led is mutable; can be scheduled Infinite loop



```
blink led =
loop
after ms 50,
led <- not (deref led)
wait led
```

led is mutable; can be scheduled Infinite loop

Schedule a future update

































led is mutable; can be scheduled Infinite loop Schedule a future update Wait for a write on a variable



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blink led =
loop
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led is mutable; can be scheduled Infinite loop Schedule a future update

Wait for a write on a variable



led is mutable; can be scheduled Infinite loop

Schedule a future update

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wait led
```

led is mutable; can be scheduled Infinite loop Schedule a future update

Wait for a write on a variable



led is mutable; can be scheduled Infinite loop

Schedule a future update

Wait for a write on a variable



SSM: Parallel Composition

A desired SSM library: input debounce

Nervous rats often jitter before making a decision; want a library that discards "on" events shorter than x ms

 \Rightarrow Parallel composition?



Feedback loops?

Simultaneous events?

Contradictions?

Simultaneous Events

What should we do with simultaneous events?

We could simply legistate them away at the input, but they are easy to generate internally.



What should this do?

Simultaneous Events

What should we do with simultaneous events?

We could simply legistate them away at the input, but they are easy to generate internally.



Seems reasonable: output is double the input

Simultaneous Events

What should we do with simultaneous events?

We could simply legistate them away at the input, but they are easy to generate internally.



Should this be allowed? What should its output be?

add2 x = x <- deref x + 2 // Add 2 as a side-effect

mult4 x = x <- deref x * 4 // Multiply by 4 as a side-effect</pre>

add2 x = x <- deref x + 2 // Add 2 as a side-effect

mult4 x = x <- deref x * 4 // Multiply by 4 as a side-effect</pre>

main =

let a = new 1 // Allocate a new mutable variable

add2 x = x <- deref x + 2 // Add 2 as a side-effect

mult4 x = x <- deref x * 4 // Multiply by 4 as a side-effect</pre>

main =

<pre>let a = new 1</pre>	// Allocate a new mutable variable					
par add2 a	// Runs first: $a \leftarrow 1 + 2 = 3$					
mult4 a	// Runs second: $a \leftarrow 3 \times 4 = 12$					

add2 x = x <- deref x + 2 // Add 2 as a side-effect

mult4 x = x <- deref x * 4 // Multiply by 4 as a side-effect</pre>

main =

let	a = new 1	// Allocate a new mutable variable				
par	add2 a mult4 a	// Runs first: $a \leftarrow 1 + 2 = 3$ // Runs second: $a \leftarrow 3 \times 4 = 12$				
par	mult4 a add2 a	// Runs third: $a \leftarrow 12 \times 4 = 48$ // Runs fourth: $a \leftarrow 48 + 2 = 50$				

Concurrent Code May Block on wait

```
blink led period =
                                11 void/unit scheduled variable
  let timer = new ()
  loop
    led <- not (deref led) // Toggle led now</pre>
    after period, timer <- () // Wait for the period
    wait timer
main led =
  par blink led (ms 50)
      blink led (ms 30)
                                // led toggles three times at time 600
      blink led (ms 20)
```

FDL 2020: C API for SSM Runtime

Basic trick: Two priority queues

First queue for scheduled variable update events, prioritized by time

Second queue for code to be executed in the current instant; prioritized by structure

A wait statement reminds the variable that something is waiting on it

When a variable is written, it schedules the waiting code in the second queue

An *after* statement deletes any existing outstanding event for the variable before scheduling a new one

FDL 2020: C API for SSM Runtime

// Variable management

```
void initialize_type(cv_type_t *var, type val)  // new
void assign_type(cv_type_t *var, uint32_t priority, type val)  // <-
void later_type(cv_type_t *var, uint64_t time, type val)  // after
bool event_on(cv_t *var)
```

// Trigger management (for wait statements)

```
void sensitize(cv_t *var, trigger_t *trigger)
void desensitize(trigger_t *trigger)
```

FDL 2020: C API Example

```
rar_examp_t *enter_examp(rar_t *caller, uint32_t priority, unit8_t depth, cv_int_t *a) {
 rar_examp_t *rar = (rar_examp_t *)
    enter(sizeof(rar_examp_t), step_examp, caller, priority, depth);
                                                                                         examp a =
                                              // Store pass-by-reference argument
 rar ->a = a;
                                                                                            let loc = new 0
 rar->trig1.rar = (rar_t *) rar;
                                              // Initialize our trigger
                                                                                            wait a
void step examp(rar t *gen rar) {
 rar examp t *rar = (rar examp t *) gen rar:
                                                                                            10c < -42
 switch (rar->pc) {
 case 0.
                                                                                            after ms 10, a <- 43
   initialize_int(&rar->loc, 0);
                                             // let loc = new 0
   sensitize((cv_t *) rar->a, &rar->trig1);
                                              // wait a
                                                                                            par foo 42 loc
   rar->pc = 1: return:
 case 1.
                                                                                            par foo 40 loc
   if (event_on((cv_t *) rar->a)) {
                                             // if @a then
     desensitize(&rar->trig1);
                                              // De-register our trigger
                                                                                                   bar 42
   } else return:
   assign_int(&rar->loc, rar->priority, 42);
                                            // loc <- 42
                                             // after 10ms, a <- 43
   later_int(rar->a, now+10000, 43);
                                              // Single routine call: foo 42 loc
   rar - pc = 2:
   call((rar_t *) enter_foo((rar_t *) rar, rar->priority, rar->depth, 42, &rar->loc));
   return:
 case 2.
                                              // Concurrent call: par foo 40 loc: bar 42
                                              11.2 children
   { uint8_t new_depth = rar->depth - 1;
     uint32_t pinc = 1 << new_depth:</pre>
     uint32 t new priority = rar->priority:
     fork((rar_t *) enter_foo((rar_t *) rar, new_priority, new_depth, 40, &rar->loc));
     new_priority += pinc:
     fork((rar t *) enter bar((rar t *) rar. new priority. new depth, 42)); }
   rar->pc = 3; return;
 case 3: : }
 leave((rar t *) rar. sizeof(rar examp t));
                                             // Terminate
```

TCRS 2023: SSM as a Lua Library

```
local ssm = require("ssm")
```

```
function ssm.pause(d)
    local t = ssm.Channel {}
    t:after(ssm.msec(d), { go = true })
    ssm.wait(t)
end
```

```
function ssm.fib(n)
  if n < 2 then
    ssm.pause(1)
    return n
  end
  local r1 = ssm.fib:spawn(n - 1)
  local r2 = ssm.fib:spawn(n - 2)
  local rp = ssm.pause:spawn(n)
  ssm.wait { r1, r2, rp }
  return r1[1] + r2[1]
  end</pre>
```

```
local n = 10
```

```
ssm.start(function()
    local v = ssm.fib(n)
```

print(("fib(%d) => %d"):format(n, v))
--- prints "fib(10) => 55"

MEMOCODE 2023: The RP2040

2 ARM Cortex M0+ processor cores, 133 MHz

264K SRAM

Off-chip QSPI flash (e.g., 2 MB)

30 GPIO pins

2 Programmable I/O Blocks (PIO)

US\$1 quantity 1



MEMOCODE 2023: A PIO Block

4 "State Machines"

32-instruction memory (shared)

9 instructions (jump, wait, in, out, etc.)

4 32-bit registers

Single-cycle execution



MEMOCODE 2023: Sslang on an RP2040



Latency: 10-20 µs Accuracy: 62.5 ns / 16 MHz

```
sleep delay =
 let timer = new ()
  after delay, timer <- ()
 wait timer
waitfor var value =
 while deref var != value
   wait var
debounce delay input press =
 loop
   waitfor input 0
   press <- ()
   sleep delay
   waitfor input 1
   sleep delav
pulse period press output =
 loop
   wait press
   output <- 1
    after period. output <- 0
   wait output
buttonpulse button led =
  let press = new ()
  par debounce (ms 10) button press
     pulse (ms 200) press led
```

21 µs Button-to-LED latency

MEMOCODE 2023: 100 µs pulse: C vs Sslang Latency



MEMOCODE 2023: 100 µs pulse: C vs Sslang Falling edge



C falling edge: 1.41 μ s late, 960 ns jitter



Sslang falling edge: 0 µs late, 62.6 ns jitter (16 MHz clock)