Choosing Determinacy: Combining Concurrency and Timing in the Sparse Synchronous Model

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Boolean Functions as a Table

For *n* Boolean inputs and *m* Boolean outputs, Each of 2*ⁿ* rows lists the *m* Boolean outputs for that row's input combination Each possible input combination appears in exactly one row

It is a total function: $2^n \rightarrow 2^m$

Acyclic Networks of NAND2 Gates

<https://www.electronics-tutorials.ws/logic/universal-gates.html>

Directed Acyclic Graph of Two-input NAND gates

Primary inputs: no incoming edges

All others: two incoming edges

Semantics: set value of each primary input; in topological order, set each node's value to the NAND of the values of its two incoming edges

Can compute any Boolean function

Deterministic: Assignment of each node's value depends only on the primary inputs, not the particular topological order chosen

Deterministic Finite Automaton as a Table

After Hopcroft and Ullman, Introduction to Automata Theory,

Languages, and Computation, 1979

- \blacktriangleright List of states, some are accepting
- ▶ A start state
- List of inputs
- Complete table of transitions (state, input) \rightarrow state

Deterministic if, for each state and input, there's exactly one next state

Synchronous Digital Logic

DAG with three types of nodes:

- ▶ NAND2: two incoming edges
- \blacktriangleright flip-flop: one incoming edge
- \triangleright primary input: no incoming edges

Every cycle in the graph must pass through a flip-flop

In each cycle, primary input nodes set to new value, flip-flop nodes set to input in last cycle (false in first)

NAND2 nodes evaluated in topological order, ignoring flop-flop input edges

Turing Machine

- \blacktriangleright A tape of symbols
- ▶ A head that can read and write symbols and move left or right
- ▶ A state register

A table of instructions: (state, symbol) \rightarrow (state, symbol, left/right) Deterministic because there's exactly one thing to do at each step

expr ::= *expr expr* | λ *variable* . *expr* | *constant* | *variable* | (*expr*)

Kozen, Church-Rosser Made Easy, Fundamenta Informaticae, 103(1–4), 2010

 $two = \lambda f \cdot \lambda x \cdot f(f x)$ $\mathsf{three} = \lambda f \cdot \lambda x \cdot f \left(f \left(f \mathbf{x} \right) \right)$ five $= \lambda f \cdot \lambda x \cdot f (f (f (f (f x))))$ plus = $\lambda m.\lambda n.\lambda f.\lambda x$. *m* f (*n* f *x*)

plus three two

Expand plus

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plus three two

 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

 $β$ -reduce ($λ$ m ...) three

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 $(\lambda n.\lambda f.\lambda x.$ three $f(n f x))$ two

β-reduce (λ n ...) two (could have expanded three)

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plus three two

 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

 $(\lambda n.\lambda f.\lambda x.$ three $f(n f x))$ two

 $\lambda f \cdot \lambda x$. three *f* (two *f x*)

Expand three and beta reduce twice (could have expanded two)

expr ::= *expr expr* | λ *variable* . *expr* | *constant* | *variable* | (*expr*)

Kozen, Church-Rosser Made Easy, Fundamenta Informaticae, 103(1–4), 2010

 $two = \lambda f \cdot \lambda x \cdot f(f x)$ $\mathsf{three} = \lambda f \cdot \lambda x \cdot f \left(f \left(f \mathbf{x} \right) \right)$ five $= \lambda f \cdot \lambda x \cdot f (f (f (f (f x))))$ plus $= \lambda m \cdot \lambda n \cdot \lambda f \cdot \lambda x$. *m* f (*n* f *x*)

plus three two

 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

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 $\lambda f \cdot \lambda x$. three *f* (two *f x*)

 $λ$ *f*. $λ$ *x*. *f* (*f* (*f* (<u>two *f x*)))</u>

Expand two and beta reduce twice

expr ::= *expr expr* | λ *variable* . *expr* | *constant* | *variable* | (*expr*)

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 $\lambda f. \lambda x. f (f (f (f (f x))))$

Normal form (nothing more to do)

expr ::= *expr expr* | λ *variable* . *expr* | *constant* | *variable* | (*expr*)

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 $(\lambda m.\lambda n.\lambda f.\lambda x. m f (n f x))$ three two

 $(\lambda n.\lambda f.\lambda x.$ three $f(n f x))$ two

 $\lambda f \cdot \lambda x$. three *f* (two *f x*)

 $λ$ *f*. $λ$ *x*. *f* (*f* (*f* (<u>two *f x*)))</u>

 $\lambda f. \lambda x. f (f (f (f (f x))))$

five

This is "five"

Many reducible sub-expressions: Church-Rosser: all choices OK

$$
\left(\left(\lambda x \cdot ((\lambda w \cdot \lambda z \cdot + w z) 1) \right) ((\lambda x \cdot x x) (\lambda x \cdot x)) \right) ((\lambda y \cdot + y 1) (+ 2 3))
$$
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\lambda z
$$
\n<math display="</math>

Many reducible sub-expressions: Church-Rosser: all choices OK

$$
\left(\left(\lambda x \cdot ((\lambda w \cdot \lambda z + w z) 1) \right) ((\lambda x \cdot x x) (\lambda x \cdot x x)) \right) ((\lambda y \cdot + y 1) (+ 2 3))
$$
\n
$$
\beta
$$
-reduction is confluent\n
$$
\begin{array}{ccc}\n\lambda x & \lambda y & \lambda y \\
\lambda w & 1 & \lambda & \lambda \\
\lambda z & x & x & x + y\n\end{array}\n\right) = \begin{array}{ccc}\n\beta^* & \beta^* & \beta^* \\
M_2 & \beta^* & \beta^* \\
\lambda z & M_2 & \lambda y\n\end{array}
$$
\n
$$
\Rightarrow \text{An expression's normal form, if it exists, is unique}
$$

Kahn Process Networks

h h $g \sim$ f Z Y $T₂$ T1 X

Network of concurrent processes communicate through FIFOs

Blocking reads; non-blocking writes

Sequence of data values passed through each FIFO is deterministic

```
process f (in int u, in int v,
           out int w) {
  int i; bool b = true;
  for (;;) {
    i = b ? wait(u) : wait(w);
    printf ("%i\n', i);send(i, w);
    b = !b:
  }
}
process g(in int u, out int v,
          out int w) {
  int i; bool b = true;
  for (;;) {
    i = \text{wait}(u);
    if (b) \text{send}(i, v);
    else send(i, w);
    b = !b;}
}
                                   process h(in int u, out int v,
                                              int init ) {
                                     int i ;
                                     send(v, init );
                                     for (;;) {
                                       i = \text{wait}(u);
                                       send(i, v);
                                     }
                                   }
                                  channel int X, Y, Z, T1, T2;
                                   f(Y, Z, X);q(X, T1, T2);
                                  h(T1, Y, 0);
                                   h(T2, Z, 1);
```
Discrete-Event Simulation: Verilog

Discrete-Event Simulation: Verilog

- 1. Select, remove, and execute earliest pending event *e* from queue
- 2. At an event @(), mark successor as sensitive
- 3. On assignment $v =$, schedule all events sensitive to the variable
- 4. On delay #, schedule successor in the future

Nondeterminism in Verilog

```
module race;
  reg a;
  initial begin #10; a = 1;
                #10; a = 0;
                #10; a = 1; end
                                                        10 first
                                                        10 second
                                                        20 second
                                                        20 first
                                                        30 first
                                                        30 second
```
always @(a) **\$display**("%0t first", \$time);

```
always @(a) $display("%0t second", $time);
```
endmodule

Adam Kepecs, Cold Spring Harbor Laboratory [Lak et al., Neuron 84(1), 2014]

Bpod: An Open Hardware Platform for Behavioral Monitoring and Control

Sanworks.io, spun out of Kepecs' lab. Teensy 3.6: ARM Cortex M4, 180 MHz

SSM: The Idea


```
let timeout = new @valve < -1delay (ms 100)
valve < -0after (s 10), timeout <- 1
wait gate || timeout
if updated timeout
 failed <- failed + 1
else
 led < -1after (ms 100), led <- 0
 wait led
```
SSM: Wishlist

Deterministic formal semantics

Explicit model-time delays only; platform-independent timing above some minimum delay (synchronous logic)

"Bare metal" microcontroller implementations: hardware counter/timer drives timing, timer interrupts for scheduling

Concurrency

Time modeled arithmetically Time in seconds

Can add, subtract, multiply, and divide time intervals

Time modeled arithmetically

Time is quantized; quantum not user-visible Quantum might be 1 MHz, 16 MHz, etc. Integer timestamps thwart Zeno

Time modeled arithmetically

Time is quantized; quantum not user-visible Program thinks processor is infinitely fast: execution a sequence of zero-time instants (hence "synchronous")

Every instruction that runs in an instant sees the same timestamp

Time modeled arithmetically

Time is quantized; quantum not user-visible Program thinks processor is infinitely fast: execution a sequence of zero-time instants (hence "synchronous")

Nothing happens in most instants (hence "sparse")

0ms 50ms 100ms 150ms

```
blink led = led is mutable; can be scheduled
 loop
   after ms 50,
     led <− not (deref led)
   wait led
```


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```
blink \text{led} =loop
    after ms 50,
       led <− not (deref led)
    wait led
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Schedule a future update

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Wait for a write on a variable


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Wait for a write on a variable

blink led $=$ **loop after** ms 50, led <− not (deref led) **wait** led

led is mutable; can be scheduled Infinite loop

Schedule a future update

Wait for a write on a variable

SSM: Parallel Composition

A desired SSM library: input debounce

Nervous rats often jitter before making a decision; want a library that discards "on" events shorter than *x* ms

⇒ Parallel composition?

Feedback loops?

Simultaneous events?

Contradictions?

Simultaneous Events

What should we do with simultaneous events?

We could simply legistate them away at the input, but they are easy to generate internally.

What should this do?

Simultaneous Events

What should we do with simultaneous events?

We could simply legistate them away at the input, but they are easy to generate internally.

Seems reasonable: output is double the input

Simultaneous Events

What should we do with simultaneous events?

We could simply legistate them away at the input, but they are easy to generate internally.

Should this be allowed? What should its output be?

add2 x = x <- **deref** x + 2 */ / Add 2 as a side-effect*

mult4 $x = x \leq -$ deref $x * 4$ // Multiply by 4 as a side-effect

add2 x = x <- **deref** x + 2 */ / Add 2 as a side-effect*

mult4 $x = x \leq$ **deref** $x * 4$ // *Multiply by 4 as a side-effect*

 $main =$

let a = **new** 1 */ / Allocate a new mutable variable*

add2 x = x <- **deref** x + 2 */ / Add 2 as a side-effect*

mult4 $x = x \leq$ **deref** $x * 4$ // *Multiply by 4 as a side-effect*

 $main =$ **let** a = **new** 1 */ / Allocate a new mutable variable* **par** add2 a *// Runs first: a* \leftarrow 1 + 2 = 3 mult4 a *// Runs second: a* \leftarrow 3 \times 4 = 12

add2 x = x <- **deref** x + 2 */ / Add 2 as a side-effect*

mult4 $x = x \leq -$ deref $x * 4$ // Multiply by 4 as a side-effect

 $main =$

Concurrent Code May Block on *wait*

```
blink led period =
  let timer = new () / / void/unit scheduled variable
  loop
    led <- not (deref led) / / Toggle led now
   after period, timer <- () / / Wait for the period
    wait timer
main \text{led }=par blink led (ms 50)
      blink led (ms 30)
      blink led (ms 20) / / led toggles three times at time 600
```
FDL 2020: C API for SSM Runtime

Basic trick: Two priority queues

First queue for scheduled variable update events, prioritized by time

Second queue for code to be executed in the current instant; prioritized by structure

A *wait* statement reminds the variable that something is waiting on it

When a variable is written, it schedules the waiting code in the second queue

An *after* statement deletes any existing outstanding event for the variable before scheduling a new one

FDL 2020: C API for SSM Runtime

```
/ / Routine activation record management
rar_t *enter(size_t size, void (*step)(rar_t *), rar_t *caller,
             uint32_t priority, uint8_t depth)
void call(rar_t *rar)
void fork(rar_t *rar)
void leave(rar_t *rar, size_t size)
```
/ / Variable management

void initialize_type(cv_type_t *var, type val) */ / new* **void** assign_type(cv_type_t *var, uint32_t priority, type val) */ / <* **void** later_type(cv_type_t *var, uint64_t time, type val) */ / after* **bool** event_on(cv_t *var)

/ / Trigger management (for wait statements)

```
void sensitize(cv_t *var, trigger_t *trigger)
void desensitize(trigger_t *trigger)
```
FDL 2020: C API Example

```
rar_examp_t *enter_examp(rar_t *caller, uint32_t priority, unit8_t depth, cv_int_t *a) {
 rar_examp_t *rar = (rar_examp_t *)
     enter(sizeof(rar_examp_t), step_examp, caller, priority, depth);
 rar->a = a;<br>
rar->trig1.rar = (rar t *) rar:<br>
// lnitialize our trigger (rar t *) rar:<br>
// lnitialize our trigger
 rar->trig1.rar = (rar t *) rar;
}
void step examp(rar t *gen_rar) {
 rar examp t *rar = (rar examp t *) gen rar;
  switch (rar->pc) {
  case 0:
    initialize\_int(Rar->loc, 0); // let loc = new 0<br>sensitize((cy t *) rar->a Rrar->tris(1) // wait a
    sensitize((cv_t *) rar->a, &rar->trig1); /
    rar->pc = 1; return;
  case 1:
    if (event_on((cv_t *) rar->a)) { // if @a then<br>desensitize(&rar->trig1): // De-register our trigger
      desensitize(&rar->trig1); / / De-register our trigger
    } else return;
    assign_int(&rar->loc, rar->priority, 42); / / loc <- 42
    later int(rar->a, now+10000, 43);
    rar->pc = 2; / / Single routine call: foo 42 loc
    call((rar t *) enter foo((rar t *) rar, rar->priority, rar->depth, 42, &rar->loc));
    return;
  case 2: // Concurrent call: par foo 40 loc; bar 42<br><i>// 2 children call: par foo 40 loc; bar 42
    { uint8_t new_depth = rar->depth - 1; / / 2 children
      uint32 t pinc = 1 \le new depth:
      uint32 t new priority = rar->priority;
      fork((rar_t *) enter_foo((rar_t *) rar, new_priority, new_depth, 40, &rar->loc));
      new_priority += pinc;
      fork((rar_t *) enter_bar((rar_t *) rar, new_priority, new_depth, 42)); }
    rar->pc = 3; return;
  case 3: ; }
  leave((rar_t *) rar, sizeof(rar_examp_t)); / / Terminate
}
```
examp $a =$ **let** $\log_2 P$ **new** θ **wait** a $loc < -42$ **after** ms 10, a <- 43 **par** foo 42 loc **par** foo 40 loc bar 42

TCRS 2023: SSM as a Lua Library

```
local ssm = require("ssm")
```

```
function ssm.pause(d)
  \text{local } t = \text{ssm}.\text{Channel } \{\}t:after(ssm.msec(d), { go = true })
  ssm.wait(t)
end
```

```
function ssm.fib(n)
  if n < 2 then
    ssm.pause(1)
    return n
  end
  local r1 = ssm. fib: spawn(n - 1)local r2 = ssm. fib: sbam(n - 2)local rp = ssm.pause:spawn(n)
  ssm.wait { r1, r2, rp }
  return r1[1] + r2[1]
  end
```

```
local n = 10
```

```
ssm.start(function()
 local v = ssm.find(n)
```
 $print('"\text{fib}(\%d) \Rightarrow \%d")$: format(n, v)) *prints "fib(10) => 55"*

local t = ssm.as_msec(**ssm.now**()) **print**(("Completed in %.2fms"):format(t)) *prints "Completed in 10.00ms"* **end**)

MEMOCODE 2023: The RP2040

2 ARM Cortex M0+ processor cores, 133 MHz

264K SRAM

Off-chip QSPI flash (e.g., 2 MB)

30 GPIO pins

2 Programmable I/O Blocks (PIO)

US\$1 quantity 1

MEMOCODE 2023: A PIO Block

4 "State Machines"

32-instruction memory (shared)

9 instructions (jump, wait, in, out, etc.)

4 32-bit registers

Single-cycle execution

MEMOCODE 2023: Sslang on an RP2040

Latency: 10-20 µs Accuracy: 62.5 ns / 16 MHz

```
let timer = new ()
 after delay, timer <- ()
 wait timer
waitfor var value =
 while deref var != value
   wait var
debounce delay input press =
 loop
   waitfor input 0
   press \leftarrow ()
   sleep delay
   waitfor input 1
    sleep delay
pulse period press output =
 loop
   wait press
   output <- 1
   after period, output <- 0
   wait output
buttonpulse button led =
 let press = new ()
 par debounce (ms 10) button press
      pulse (ms 200) press led
```
sleep delay =

21 µs Button-to-LED latency

MEMOCODE 2023: 100 µs pulse: C vs Sslang Latency

MEMOCODE 2023: 100 µs pulse: C vs Sslang Falling edge

C falling edge: 1.41 µs late, 960 ns jitter

Sslang falling edge: 0 µs late, 62.6 ns jitter (16 MHz clock)