

COMS 6998 Proof Complexity and Applications, Spring 2025

Project Ideas

Below are some ideas for your paper/project presentation. I have categorized roughly by topic. There are too many papers to list them all in each topic, so once you pick a topic you may want to discuss with me and also do a search to find other related papers. Also you are free to choose a topic outside of this list provided that you discuss with me first. We will set up a time for you to discuss with me and Tianqi ahead of your presentation.

You may work on your project in small groups, ideally one or two people, but possibly more depending on the topic.

Project Proposal Submission. You should submit the topic/paper(s) that you will present and your group members via email (with subject "Project Topic") by February 26th. If more than one group selects the same topic, I will give it to the first group to submit.

1. Space Complexity and Time-Space-Depth Tradeoffs

- (a) (*) Nordstrom. Pebble Games, Proof Complexity and Time-Space Tradeoffs.
<https://people.csail.mit.edu/jakobn/research/LMCSsurvey.pdf>
- (b) Beame, Beck, Impagliazzo. Time-space tradeoffs in resolution: superpolynomial lower bounds for superlinear space. STOC 2012
<https://homes.cs.washington.edu/~beame/papers/proofspace-journal.pdf>
- (c) Bonacina, Galesi, Thapen. Total space in resolution.
<https://epubs.siam.org/doi/abs/10.1137/15M1023269>
- (d) (*) Razborov. A New Kind of Tradeoff in Propositional Proof Complexity
<http://people.cs.uchicago.edu/~razborov/files/ultimate.pdf>

2. Proof Complexity, SOS and Hardness of Approximation

- (a) Buresh-Oppenheimer, Galesi, Hoory, Magen, Pitassi. Rank Lower Bounds and Integrality Gaps for the Cutting Planes Procedure, 2006. <https://ieeexplore.ieee.org/document/1238206>
- (b) (*) Fleming, Kothari, Pitassi. Semi-algebraic Proofs and Efficient Algorithm Design.
<https://www.nowpublishers.com/article/Details/TCS-086>

3. Proof Complexity and Total Search classes

- (a) (*) Buss, Fleming, Impagliazzo. TFNP characterization of proof systems and monotone circuits (2023)
<https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.ITCS.2023.30>
- (b) Buresh-Oppenheimer, Morioka. Relativized NP search problems and propositional proof systems.
<https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=1313795>
- (c) Goos, Kamath, Robere. Adventures in monotone complexity and TFNP.
<https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.ITCS.2019.38>
- (d) (*) de Rezende, Goos, Robere Proofs, Circuit and Communication, SIGACT news 2022.
<https://arxiv.org/abs/2202.08909>
- (e) E. Jerebik. Dual weak pigeonhole principle, Boolean complexity and derandomization.
<https://www.sciencedirect.com/science/article/pii/S0168007204000156>
- (f) E. Jerabek. Approximate counting and bounded arithmetic.
<https://www.jstor.org/stable/pdf/27588579.pdf>
- (g) Korten, Pitassi. Strong versus weak Range Avoidance and the linear ordering principle, 2024.
<https://ieeexplore.ieee.org/abstract/document/10756044/>

4. Extended Formulations

- (a) Lee, Raghavendra, Steurer. Lower bounds on the size of semidefinite programming relaxations.
<https://arxiv.org/abs/1411.6317>
- (b) Chan, Lee, Raghavendra, Steurer. Approximate Constraint Satisfaction Requires Large LP Relaxations.
<https://dl.acm.org/doi/10.1145/2811255>

5. Automatizability, Feasible Interpolation and Connections

- (a) (*) Bonet, Pitassi, Raz. On Interpolation and Automatization for Frege Systems.
<https://www.cs.upc.edu/~bonet/revistas/siam3.pdf>
- (b) (*) Atserias, Muller. Automating Resolution is NP-hard.
<https://dl.acm.org/doi/10.1145/3409472>
- (c) Alekhovich, Braverman, Feldman, Klivans, Pitassi. Learnability and automatizability, 2004.
<https://mbraverm.princeton.edu/files/learnabilityAutomatizability.pdf>

6. Algebraic Proof Complexity

- (a) (*) Clegg, Edmonds and Impagliazzo. Using the Groebner basis algorithm for find proofs of unsatisfiability. STOC 1996.
<https://dl.acm.org/doi/10.1145/237814.237860>
Defines Polynomial calculus, gives automating algorithm. A classic, great paper.
- (b) (*) Beame, Impagliazzo, Krajicek, Pitassi, Pudlak. Lower bounds on Hilbert's Nullstellensatz and propositional proofs.
<https://homes.cs.washington.edu/~beame/papers/nsatz.pdf>
Original paper that defines the Nullstellensatz propositional proofs.
- (c) Pitassi. Algebraic Propositional Proof Systems.
<https://drops.dagstuhl.de/storage/00lipics/lipics-vol198-icalp2021/LIPIcs.ICALP.2021.5/LIPIcs.ICALP.2021.5>
The original paper that defines algebraic proof systems.
- (d) (*) Pitassi, Tzameret. Algebraic Proof Complexity: Progress, Frontiers and Challenges. SigLog News.
<https://arxiv.org/pdf/1607.00443.pdf>
- (e) (*) Grochow, Pitassi. Circuit Complexity, Proof Complexity and the Ideal Proof System.
<https://dl.acm.org/doi/10.1145/3230742>
Defines IPS and proves that IPS lower bounds imply algebraic circuit lower bounds. This is the first paper to connect proof lower bounds to circuit lower bounds.
- (f) Semi-Algebraic proofs, IPS lower bounds and the tau-conjecture.
<https://dl.acm.org/doi/abs/10.1145/3357713.3384245>
- (g) Proof complexity lower bounds from algebraic circuit complexity (2021)
<https://theoryofcomputing.org/articles/v017a010/>
- (h) Ideals, determinants and straightening: Proving and using lower bounds for polynomial ideals, 2022.
<https://dl.acm.org/doi/abs/10.1145/3519935.3520025>
- (i) Functional Lower Bounds to Algebraic Proofs: Symmetry, Lifting and Barriers, STOC 2024.
<https://dl.acm.org/doi/pdf/10.1145/3618260.3649616>

7. Resolution, $\text{Res}(k)$, and $\text{Res}(\oplus)$ Lower Bounds

- (a) (*) Ben-Sasson and Wigderson. Short Proofs are Narrow - Resolution made simple.
<https://dl.acm.org/doi/10.1145/375827.375835>

- (b) Razborov. Pseudorandom generators hard for k-DNF Resolution and Polynomial Calculus Resolution.
<http://annals.math.princeton.edu/2015/181-2/p01>
- (c) (*) Alekhovich, Ben-Sasson, Wigderson. Pseudorandom Generators in Proof Complexity.
<http://people.cs.uchicago.edu/>
- (d) A Switching Lemma for Small Restrictions and Lower Bounds for kDNF Resolution.
<https://ieeexplore.ieee.org/document/1181984>
- (e) Bhattacharya, Chattopadhyay, Dvorak. Exponential Separation between Powers of Regular and General Resolution over parities.
<https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.CCC.2024.23>

8. Bounded-Depth Frege Lower Bounds

- (a) Ajtai. The Complexity of the Pigeonhole Principle
<https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=arnumber=21951tag=1>
- (b) Beame, Cook, Impagliazzo. Exponential Lower Bounds for the pigeonhole principle.
<https://link.springer.com/article/10.1007/BF01200117>
See also Krajicek, Pudlak, Woods.
<https://onlinelibrary.wiley.com/doi/abs/10.1002/rsa.3240070103>
- (c) (*) Urquhart, Fu. Simplified Lower Bounds for Propositional Proofs
<https://philpapers.org/rec/ALASLB>
- (d) Hastad. On small-depth Frege proofs for Tseitin grids, 2020.
<https://dl.acm.org/doi/abs/10.1145/3425606>

9. Upper Bounds in Proof Complexity

- (a) Paris, Woods, Wilkie. Provability of the Pigeonhole principle and existence of infinitely many primes.
<https://www.jstor.org/stable/2274618?seq=1>
- (b) (*) Maciel, Pitassi. A new proof of the weak pigeonhole principle, STOC 2000.
<https://dl.acm.org/doi/pdf/10.1145/335305.335348>

10. Implicit Proofs

- (a) Krajicek. Implicit Proofs.
<https://eccc.weizmann.ac.il/eccc-reports/2003/TR03-055/index.html>
- (b) Jump Operators, Interactive Proofs and Proof Complexity Generators, FOCS 2024.
<https://ieeexplore.ieee.org/abstract/document/10756097/>

11. Bounded Arithmetic and Unprovability of circuit lower bounds

- (a) Towards $P \neq NP$ from Extended Frege lower bounds
<https://arxiv.org/abs/2312.08163>
- (b) On the Correspondence between arithmetic theories and propositional proof systems- a survey.
<https://onlinelibrary.wiley.com/doi/abs/10.1002/malq.200710069>
- (c) Strong co-nondeterministic lower bounds for NP cannot be proved feasibly
<https://dl.acm.org/doi/abs/10.1145/3406325.3451117>
- (d) Unprovability of strong complexity lower bounds in bounded arithmetic
<https://dl.acm.org/doi/abs/10.1145/3564246.3585144>
- (e) On the consistency of circuit lower bounds for nondeterministic time
<https://dl.acm.org/doi/abs/10.1145/3564246.3585253>
- (f) LEARN-uniform circuit lower bounds and provability in bounded arithmetic.
<https://ieeexplore.ieee.org/abstract/document/9719862/>