COMS 6998 Proof Complexity and Applications, Spring 2025
Project Ideas

I have categorized roughly by topic. There are too many papers to list them all in each topic, so once you pick a topic you should do a search to find all related and relevant papers. I strongly encourage you to discuss your topic with me ahead of time.

Implicit Proofs.

1. Krajicek. Implicit Proofs.
   https://eccc.weizmann.ac.il/eccc-reports/2003/TR03-055/index.html

2. Khaniki, FOCS 2024.
   Jump Operators, Interactive Proofs and Proof Complexity Generators, FOCS 2024.
   https://ieeexplore.ieee.org/abstract/document/10756097/

Bounded Arithmetic and Unprovability of circuit lower bounds.

1. Pich, Santhanam. Towards P $\neq$ NP from Extended Frege lower bounds
   https://arxiv.org/abs/2312.08163

2. Beyersdorff. On the Correspondence between arithmetic theories and propositional proof systems- a survey.
   https://onlinelibrary.wiley.com/doi/abs/10.1002/malq.200710069

3. Pich, Santhanam. STOC 2021.
   Strong co-nondeterministic lower bounds for NP cannot be proved feasibly
   https://dl.acm.org/doi/10.1145/3406325.3451117

4. Chen, Li, Oliveira, FOCS 2024.
   Reverse mathematics of complexity lower bounds.
   https://eccc.weizmann.ac.il/report/2024/060/

5. Li, Oliveira, STOC 2023.
   Unprovability of strong complexity lower bounds in bounded arithmetic
   https://arxiv.org/abs/2305.15235

6. Atserias, Buss, Muller. STOC 2023.
   On the consistency of circuit lower bounds for nondeterministic time
   https://dl.acm.org/doi/abs/10.1145/3564246.3585253

7. Carmosino, Kabanets, Kolokolova, Oliveira. FOCS 2021.
   LEARN-uniform circuit lower bounds and provability in bounded arithmetic.
   https://ieeexplore.ieee.org/abstract/document/9719862/

Space Complexity and Time-Space-Depth Tradeoffs.

1. Nordstrom. Pebble Games, Proof Complexity and Time-Space Tradeoffs.
   Logical Methods in Computer Science, 9(3), article 15, Sept 2013.

2. Ben-Sasson, Nordstrom, ICS 2011.
   Understanding space in proof complexity: separations and tradeoffs via substitutions.

3. Beame, Beck, Impagliazzo, STOC 2012.
   Time-space tradeofs in resolution: superpolynomial lower bounds for superlinear space. (See also ECCC TR11-149)

4. Bonacina, Galesi, Thapen. Total space in resolution.
   SIAM Journal on Computing, Vol 45:5, 2016.

5. Razborov, An Ultimate Trade-off in Propositional Proof Complexity.
   ECCC 22:33 (2015). This paper concerns ”supercritical tradeoffs”.

6. Fleming, Pitassi, Robere, ITCS 2022. Extremely Deep Proofs.
   https://eccc.weizmann.ac.il/report/2021/158/

7. de Rezende, Fleming, Janett, Nordstrom, Pang.
   Truly Supercritical Tradeoffs for Resolution, Cutting Planes, Monotone Circuits and Weisfeiler-Lehman. https://dblp.org/db/journals/corr/corr2411.htmlabs-2411-14267
   See also Supercritical Tradeoffs for Monotone Circuits by Goos, Maystre, Risse, Sokolov.
   https://dblp.org/db/journals/corr/corr2411.htmlabs-2411-14268

Proof Complexity, SOS and Hardness of Approximation.

1. Buresh-Oppenheim, Galesi, Hoory, Magen, Pitassi. Rank Lower Bounds and Integrality Gaps for the Cutting Planes Procedure. Theory of Computing, 2, 2006, pp.65-90.

2. Arora, Bollobas, Lovasz, Tourlakis. Proving integrality gaps without knowing the linear program. Theory of Computing, Volume 2 (2006), pp. 19-51.

3. Lee, Raghavendra, Steurer. STOC 2015.
   Lower bounds on the size of semidefinite programming relaxations.

4. Lee, Raghavendra, Steurer, Tan. CCC 2014
   On the Power of Symmetric LP and SDP Relaxations.

5. Fleming, Kothari, Pitassi. Semialgebraic Proofs and Efficient Algorithm Design.
   https://eccc.weizmann.ac.il/report/2019/106/
   This is a book giving a careful treatment of semialgebraic proof systems (Sherali Adams and SOS) and their relationship to Linear and Semidefinite Programs, and algorithms as well as lower bounds for approximation algorithms for optimization problems.

Proof Complexity and Total Search classes.

1. Beame, Cook, Edmonds, Impagliazzo, Pitassi. STOC 1995.
   The relative complexity of NP search problems.
   The original paper that introduced the query complexity of TFNP; prove several separations and connections to proof complexity.

2. Buss, Fleming, Impagliazzo. ITCS 2023.
   TFNP characterization of proof systems and monotone circuits, ITCS 2023.
   https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.ITCS.2023.30

3. Buresh-Oppenheim, Morioka. CCC 2004.
   Relativized NP search problems and propositional proof systems.
   https://eccc.weizmann.ac.il/eccc-reports/2003/TR03-051/index.html

4. Goos, Kamath, Robere. Adventures in monotone complexity and TFNP.
https://drops.dagstuhl.de/entities/document/10.4230/LIPIcs.ITCS.2019.38

5. de Rezende, Goos, Robere Proofs, Circuit and Communication, SIGACT news 2022.
https://arxiv.org/abs/2202.08909

6. Goos, Hollender, Jain,Maystre, Pires, Robere, Tao. JACM 2024.
Separations in Proof Complexity and TFNP.
https://dblp.org/db/journals/jacm/jacm71.htmlGoosHJMPRT24

7. E. Jerabek. Dual weak pigeonhole principle, Boolean complexity and derandomization.
https://www.sciencedirect.com/science/article/pii/S0168007204000156

8. E. Jerabek. Approximate counting and bounded arithmetic.
https://www.cambridge.org/core/journals/journal-of-symbolic-logic/article/approximate-counting-in-bounded-arithmetic/A7B01D53C883261836B93CD036B1FE9D

9. Korten, Pitassi. FOCS 2024.
Strong versus weak Range Avoidance and the linear ordering principle, 2024.
https://ieeexplore.ieee.org/abstract/document/10756044/

Automatizability, Feasible Interpolation and Connections

1. Ben-Sasson and Wigderson. This classic paper relates the size of Resolution refutations to their width. They also give nontrivial automatizing algorithms for Resolution and tree-like Resolution.

2. Bonet, Pitassi, Raz. On Interpolation and Automatization for Frege Systems.
https://www.cs.upc.edu/ bonet/revistas/siam3.pdf
This paper defines the notion of automatizability (or automizability) of proof systems and relates them to feasible interpolation, and shows lower bounds for automizability of Frege Systems, under crypto assumptions. See references for related earlier results (Krajicek,Pudlak) that show no feasible interpolation for Extended Frege under crypto assumptions. Later papers also give conditional lower bounds for bounded-depth Frege under crypto assumtions.

3. Atserias, Muller. Automating Resolution is NP-Hard. JACM 2020.
https://dblp.org/db/journals/corr/corr1904.htmlabs-1904-02991
This is a breakthrough paper that proves NP-hardness of automizing Resolution.

4. Alekhnovich, Braverman, Feldman, Klivans, Pitassi. Learnability and automatizability, Focs 2004. Journal version: The complexity of properly learning simple concept classes. JCSS 74(1), 2008. This paper is about the connections between automatizing proof systems and learnability.

5. Atserias and Maneva. Mean-payoff games and propositional proofs. This paper connects automatizability of weak proof systems to mean payoff games.

6. Huang, Pitassi. Automatizability and Simple Stochastic Games. ICALP 2011. (Found on my homepage.) Another paper connecting automatizability but this time to simple stochastic games.

7. Beckmann, Pudlak, Thapen. Parity games and propositional proofs. ACM Transactions on Computational Logic. Volume 15:2(17), 2014. Connects automatizability to complexity of parity games.

Algebraic Proof Complexity

1. Clegg, Edmonds and Impagliazzo. Using the Groebner basis algorithm for find proofs of unsatisfiability. STOC 1996. Defines Polynomial calculus, gives quasipolynomial-time algorithm for automatization problem for poly calculus. A classic and great paper.

2. Beame, Impagliazzo, Krajicek, Pitassi, Pudlak. Lower bounds on Hilbert's Nullstellensatz and propositional proofs. FOCS 1994. (Also journal paper available.) Original paper that defines the Nullstellensatz propositional proof system, and also gives weak degree lower bounds.

3. Pitassi. Algebraic Propositional Proof Systems. 1996 Survey article (on my webpage). The original paper that defines algebraic proof systems, with many basic observations and questions.

4. Pitassi, Tzameret, Siglog News.
   Algebraic Proof Complexity: Progress, Frontiers and Challenges.
   ArXiv: https://arxiv.org/pdf/1607.00443.pdf
   This is a survey paper on algebraic proof systems.

5. Grochow, Pitassi. Circuit Complexity, Proof Complexity and the Ideal Proof System.
   JACM. This paper shows that IPS lower bounds (that is lower bounds for general algebraic proofs) implies algebraic circuit lower bounds (namely VNP not equal to VP). This is the first paper to connect circuit lower bounds to proof system lower bounds.

6. Alekseev, Grigoriev, Hirsch, Tzameret.
   Semi-Algebraic proofs, IPS lower bounds and the tau-conjecture.
   https://dl.acm.org/doi/abs/10.1145/3357713.3384245
   Conditional lower bounds for algebraic proof systems.

7. Forbes, Shpilka, Tzameret, Wigerson.
   Proof complexity lower bounds from algebraic circuit complexity (2021)
   https://theoryofcomputing.org/articles/v017a010/
   This paper gives a variety of nice lower bounds for restricted subclasses of IPS via known techniques from algebraic circuit complexity.

8. Andrews, Forbes, STOC 2022.
   Ideals, determinants and straightening: Proving and using lower bounds for polynomial ideals.
   https://dl.acm.org/doi/abs/10.1145/3519935.3520025
   Strong lower bounds for algebraic proofs (but for systems of unsolvable poly equations that don't correspond to CNF formulas).

9. Hakoniemi, Limaye, Tzameret, STOC 2024.
   Functional Lower Bounds in Algebraic Proofs: Symmetry, Lifting and Barriers, STOC 2024.
   https://dl.acm.org/doi/pdf/10.1145/3618260.3649616
   Most recent paper giving strong lower bounds for algebraic proofs.

Classic and more Recent Lower Bound Papers

1. Haken. The Intractability of Resolution, 1985.
   https://www.sciencedirect.com/science/article/pii/0304397585901446
   Breakthrough paper proving first exponential lower bounds for unrestricted resolution.

2. Ajtai. The complexity of the Pigeonhole Principle. FOCS 1988, 346-355. Breakthrough paper proving the first superpolynomial lower bounds for bounded-depth Frege proofs.

3. Beame, Cook, Impagliazzo. Exponential Lower Bounds for the pigeonhole principle.
   https://link.springer.com/article/10.1007/BF01200117
   See also Krajicek, Pudlak, Woods.
   https://onlinelibrary.wiley.com/doi/abs/10.1002/rsa.3240070103

4. Hastad. On small-depth Frege proofs for Tseitin grids, 2020.
   https://dl.acm.org/doi/abs/10.1145/3425606
   See also On bounded-depth proofs for Tseitin formulas on the grid, revisited, by Hastad, Risse.

5. Hastad. FOCS 2023. On small-depth Frege proofs for PHP.
   https://dblp.org/db/journals/corr/corr2401.htmlabs-2401-15683

6. Fleming, Pankratov, Pitassi, Robere. Random $\log n$-CNFs are Hard for Cutting Planes.
   J. ACM 69(3): 19:1-19:32 (2022)

7. Pitassi, Ramakirhsnan, Tan. FOCS 2021. Tradeoffs for small-depth Frege proofs.
   https://ieeexplore.sieee.org/document/9719723

Upper Bounds in Proof Complexity

1. Maciel, Pitassi. A new proof of the weak pigeonhole principle, STOC 2000.
   https://dl.acm.org/doi/pdf/10.1145/335305.335348
   This paper gives an improved construction of low-depth Frege proofs of the weak PHP, namely putting them in Res(polylogn). See also earlier breathrough paper by Paris, Woods and Wilkie which puts them in depth 2.5 Frege:
   Provability of the pigeonhole principle and the existence of infinitely many primes. JSL, 53, 1988.

2. Buss, Kabanets, Kolokolova, Koucky. Expander Construction in VNC1.
   https://www.sciencedirect.com/science/article/pii/S0168007220300208
   Shows that expander graphs can be provably defined in polynomial-size Frege.

3. Buss and coauthors give many interesting upper bounds in proof complexity, including Frege proofs of the PHP, the HEX tautologies, st-connectivity. See his webpage and/or discuss with me.