

Fundamentals of Computer Systems

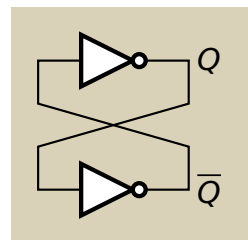
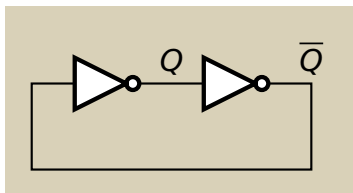
Sequential Logic

Timothy K Paine

Columbia University

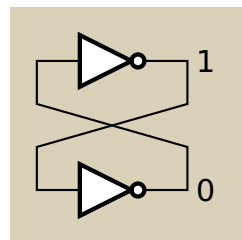
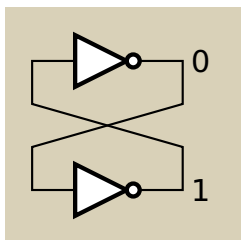
Summer 2024

Bistable Elements

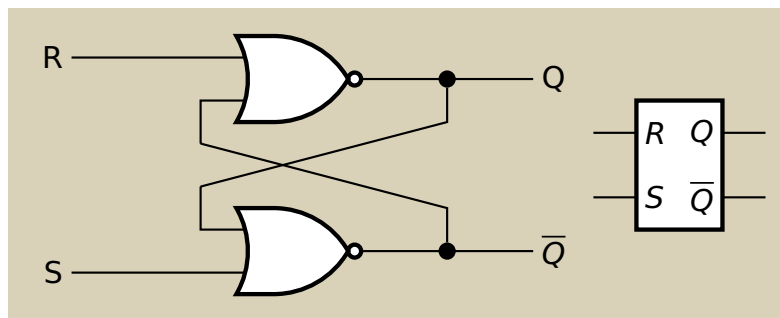


Equivalent circuits; right is more traditional.

Two stable states:

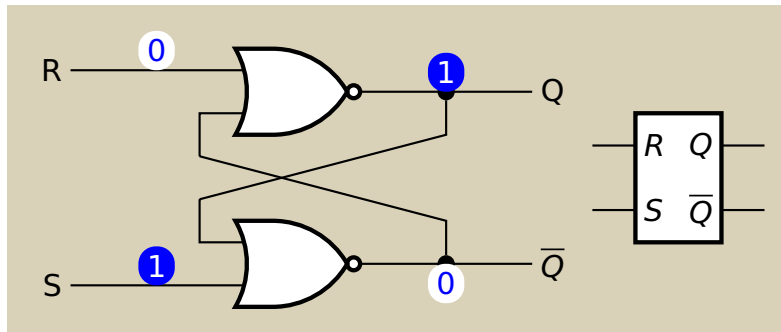


RS Latch



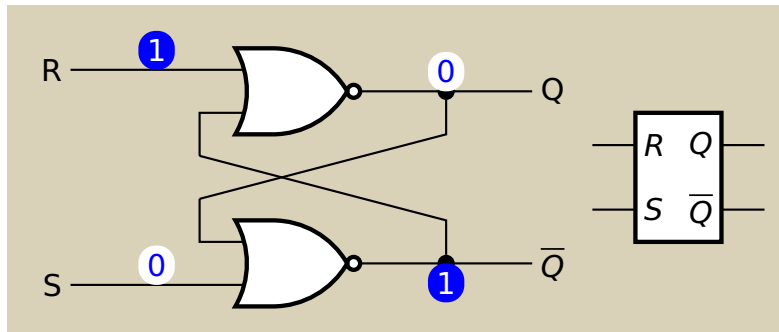
R	S	Q	\bar{Q}
0	0		
0	1		
1	0		
1	1		

RS Latch



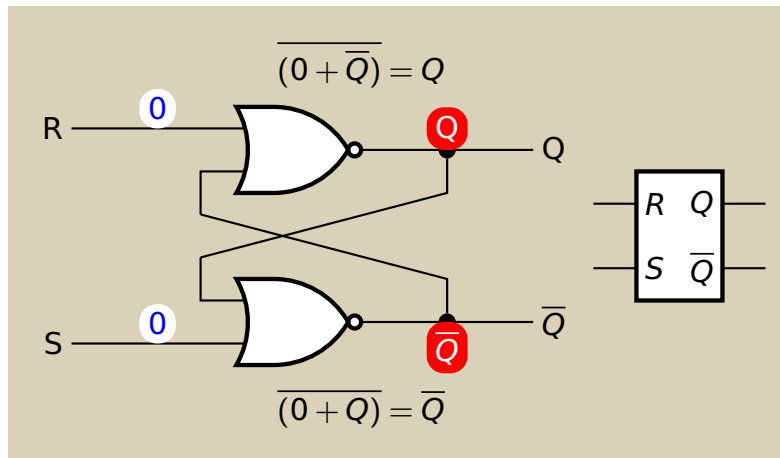
R	S	Q	\bar{Q}	
0	0			
0	1	1	0	Set ($Q = 1$)
1	0			
1	1			

RS Latch



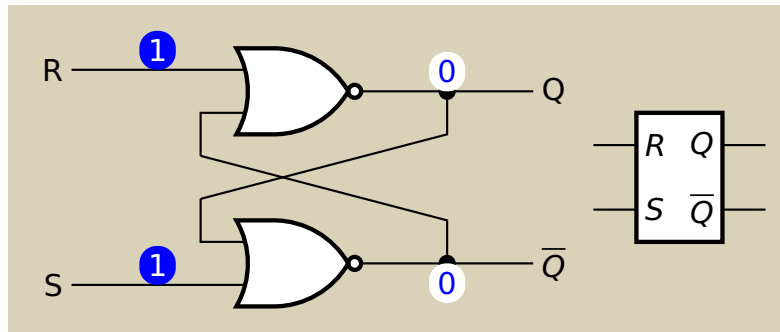
R	S	Q	\bar{Q}	
0	0			
0	1	1	0	Set ($Q = 1$)
1	0	0	1	Reset ($Q = 0$)
1	1			

RS Latch



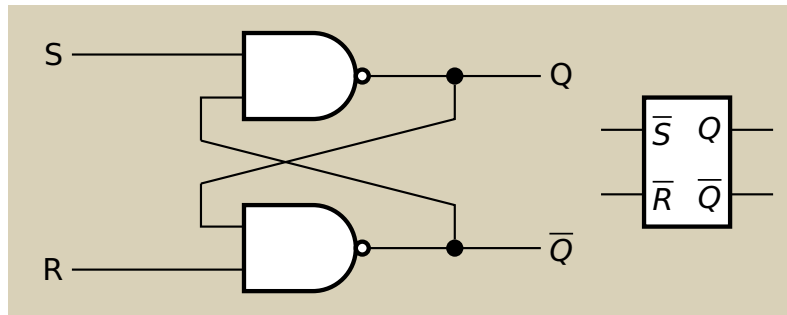
R	S	Q	\bar{Q}	
0	0	Q	\bar{Q}	Hold previous value
0	1	1	0	Set ($Q = 1$)
1	0	0	1	Reset ($Q = 0$)
1	1			

RS Latch



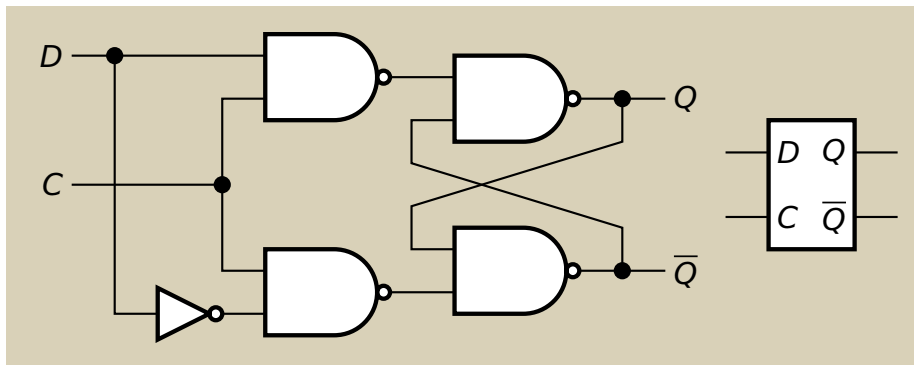
R	S	Q	\bar{Q}	
0	0	Q	\bar{Q}	Hold previous value
0	1	1	0	Set ($Q = 1$)
1	0	0	1	Reset ($Q = 0$)
1	1	0	0	Bad. Do not use.

$\overline{R}\overline{S}$ Latch



R	S	Q	\overline{Q}	
0	0	1	1	Bad. Do not use.
0	1	0	1	Reset ($Q = 0$)
1	0	1	0	Set ($Q = 1$)
1	1	Q	\overline{Q}	Hold previous value

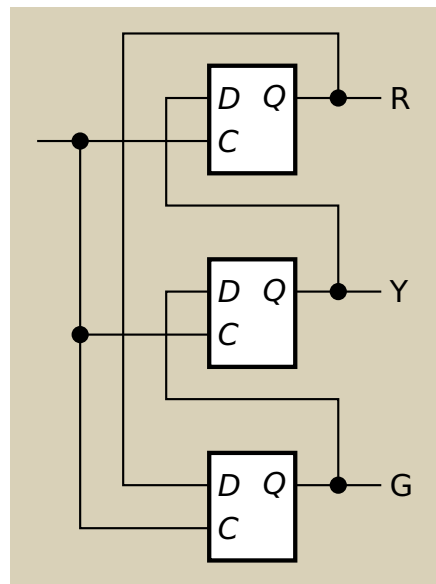
D Latch



C	D	Q	\bar{Q}
0	X	Q	\bar{Q}
1	0	0	1
1	1	1	0

A Challenge: Build a traffic light controller

Want the lights to cycle green-yellow-red.



Does this work?





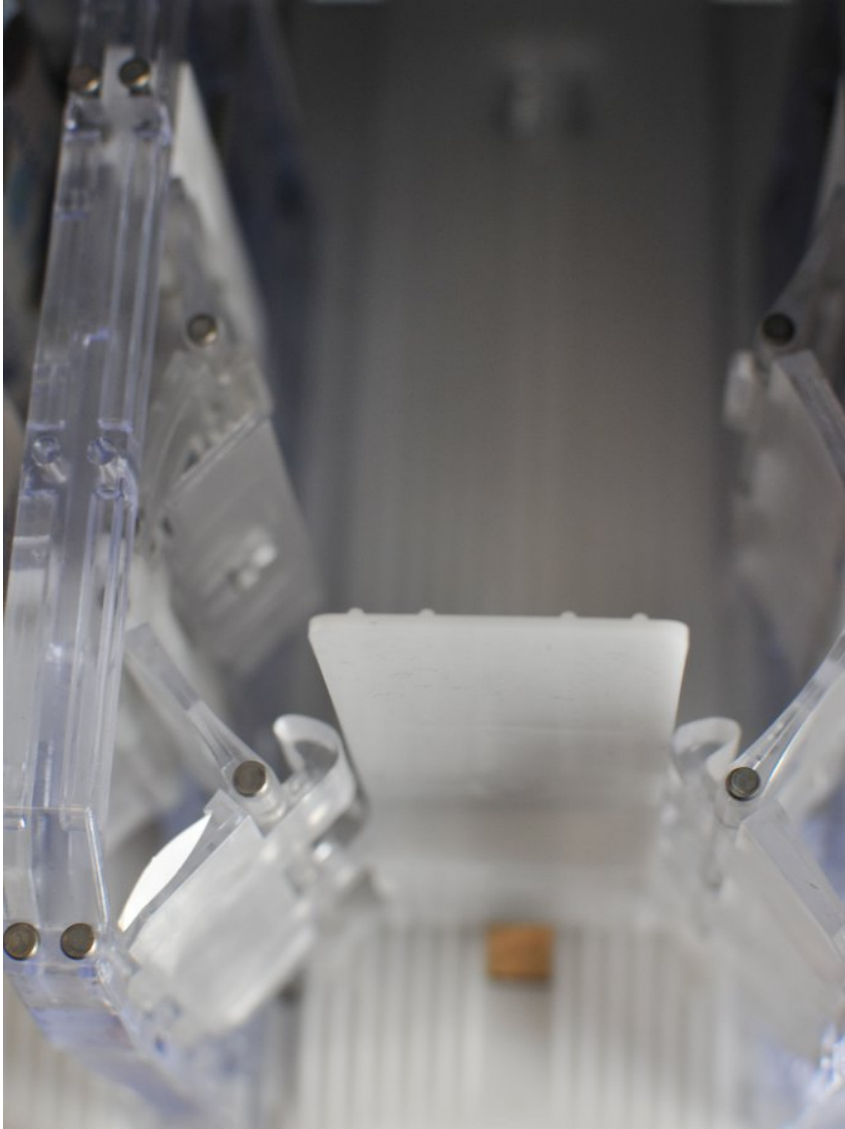


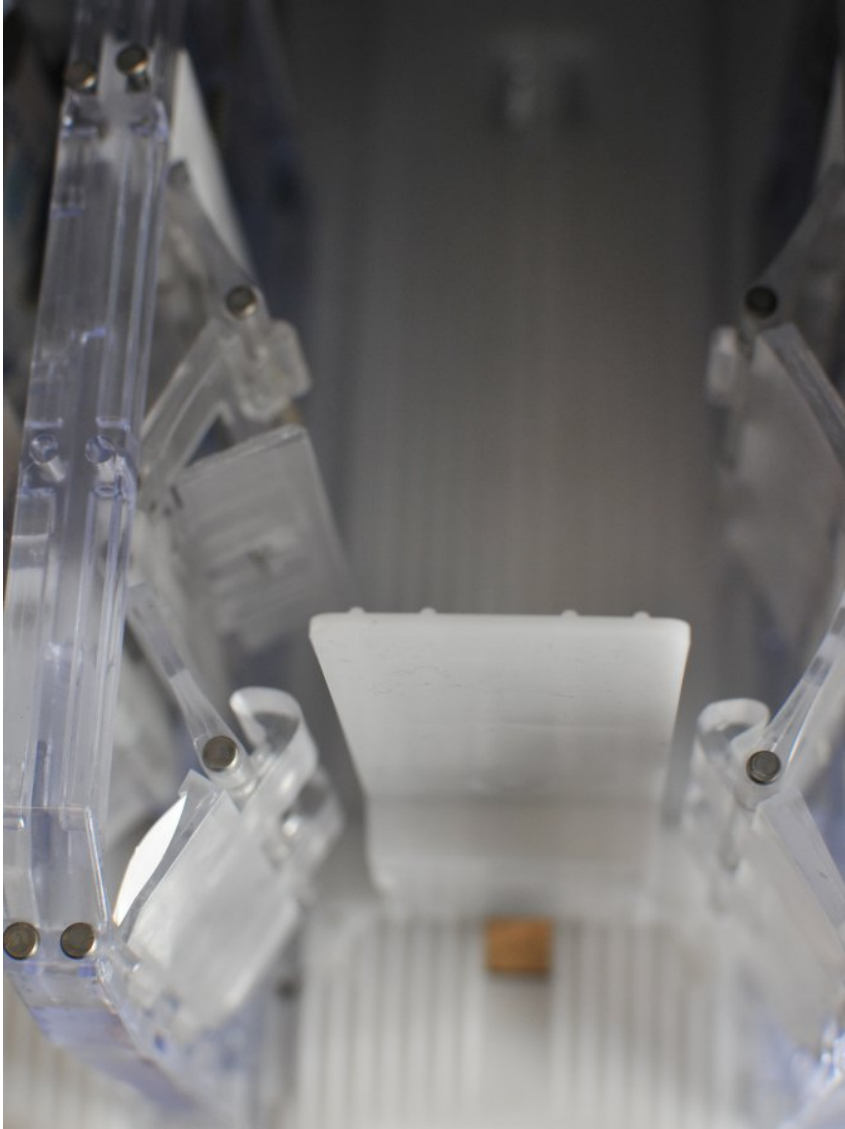


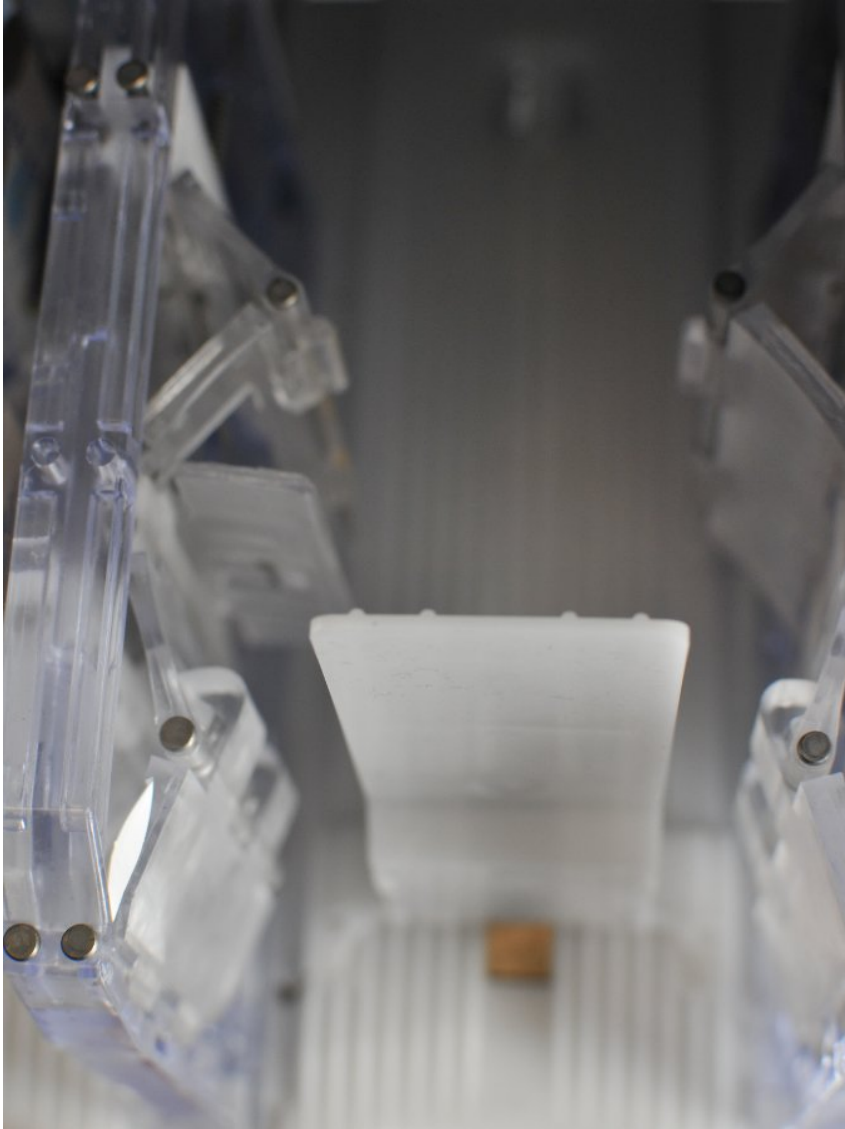


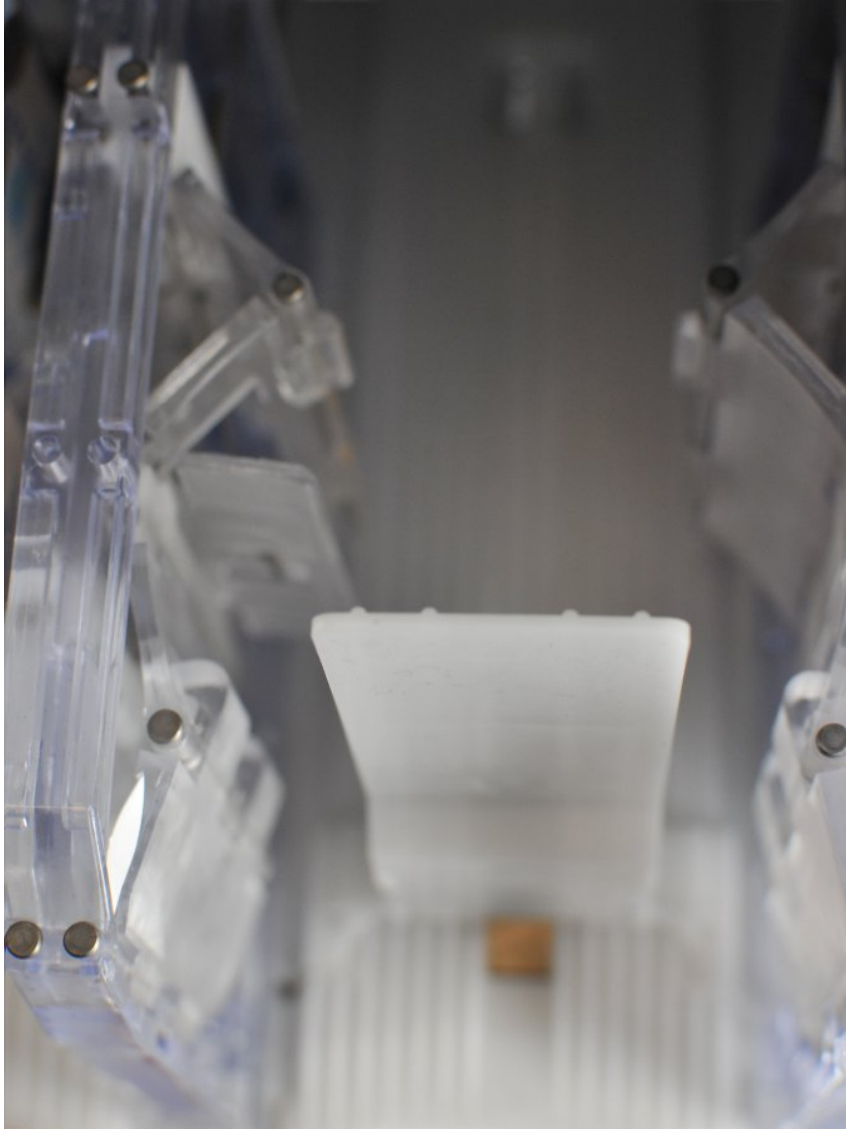


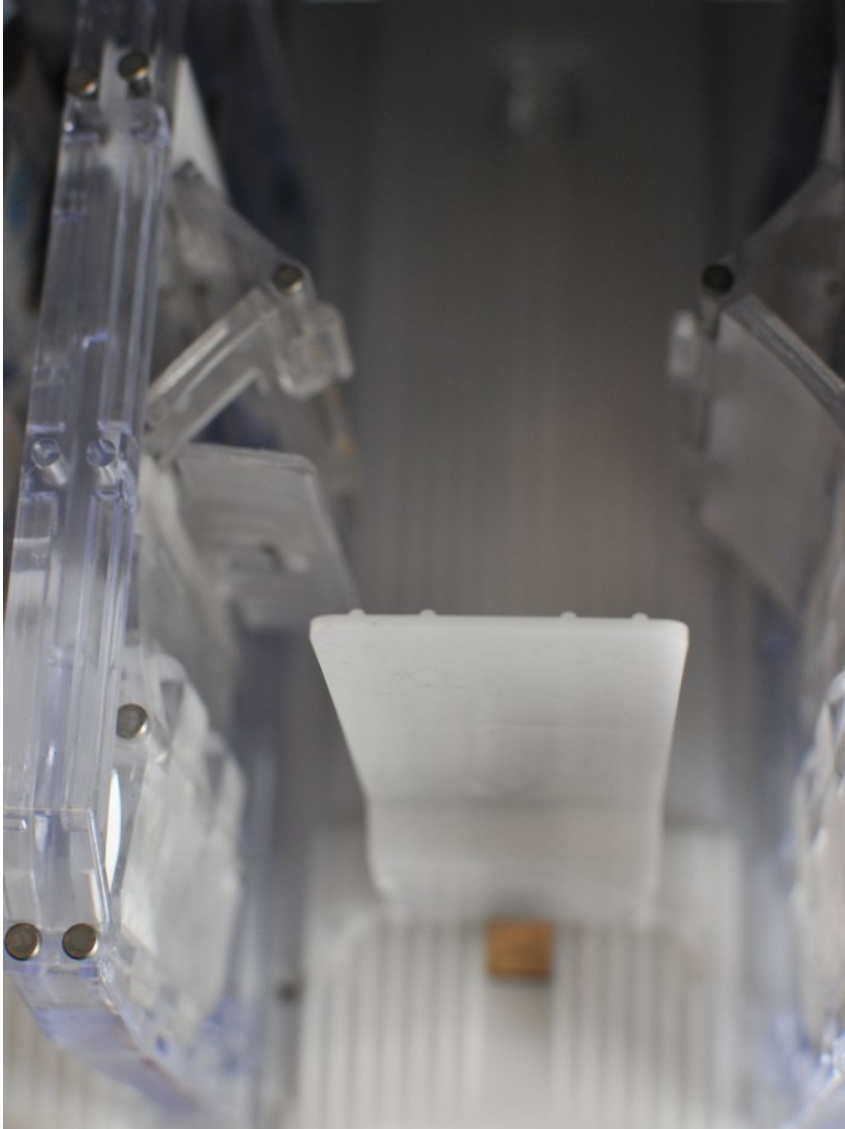




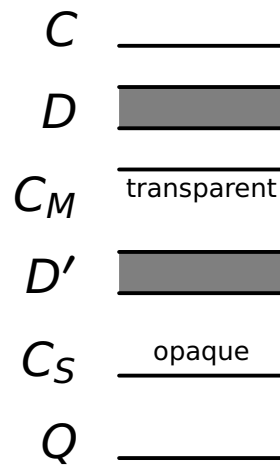
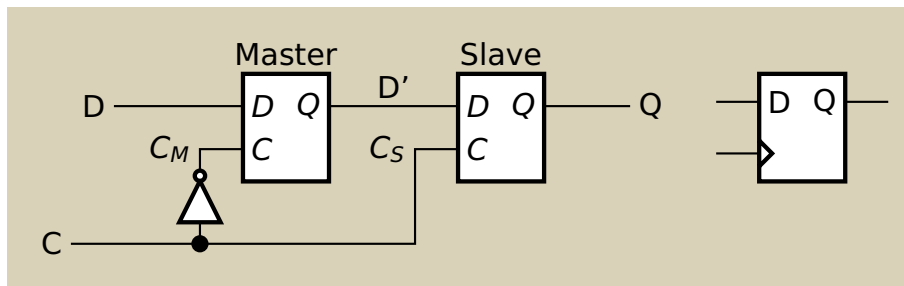




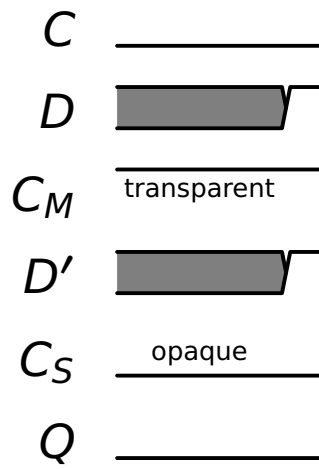
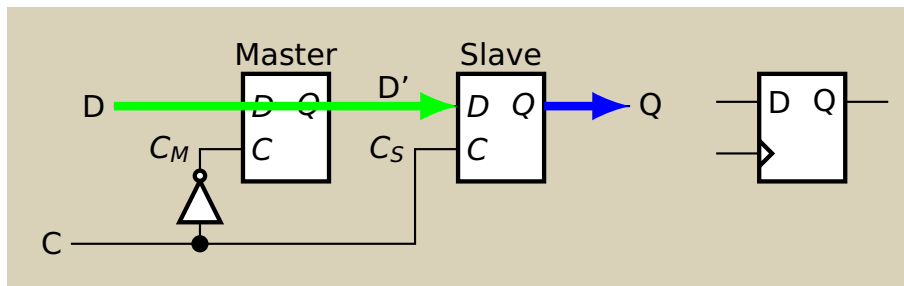




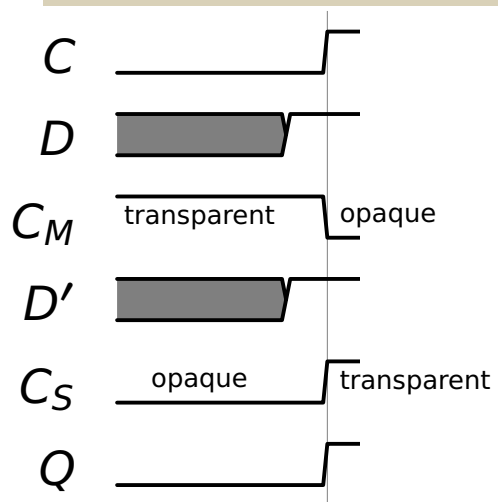
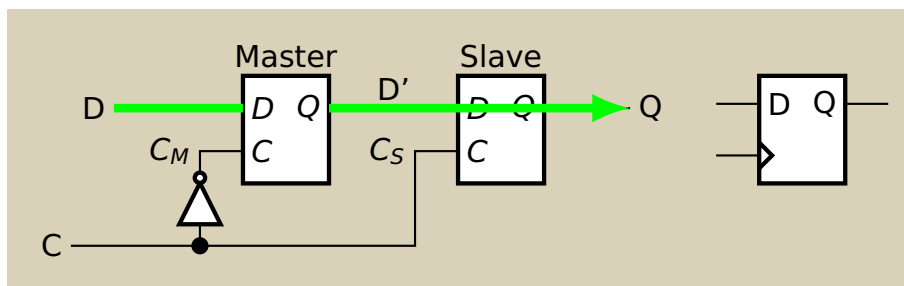
Positive-Edge-Triggered D Flip-Flop



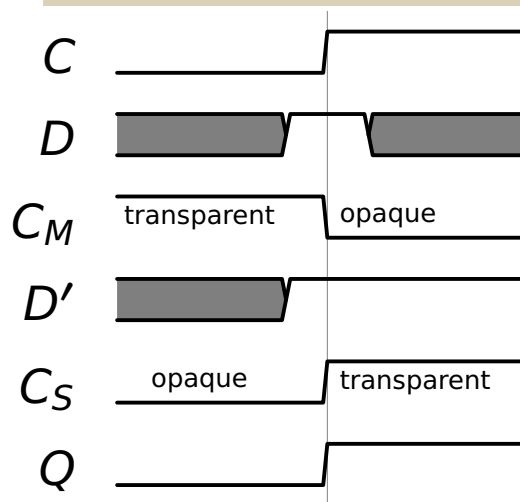
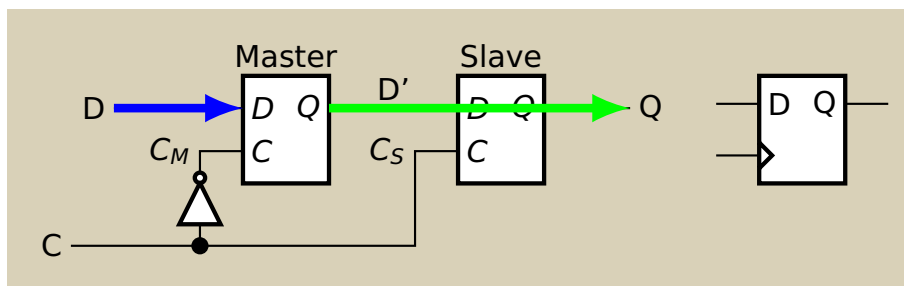
Positive-Edge-Triggered D Flip-Flop



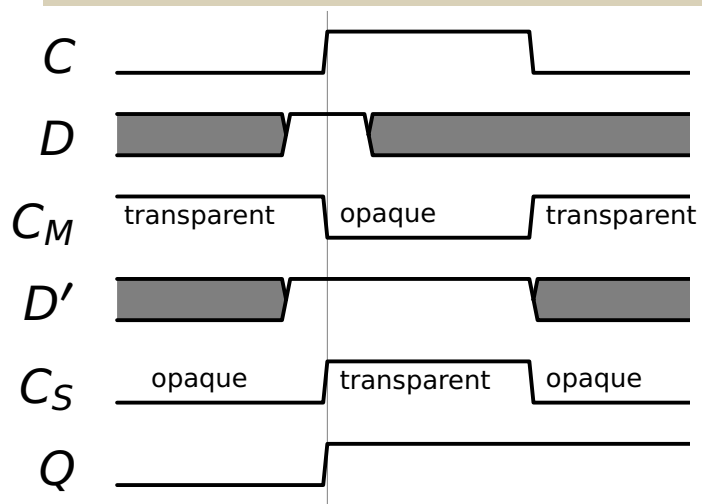
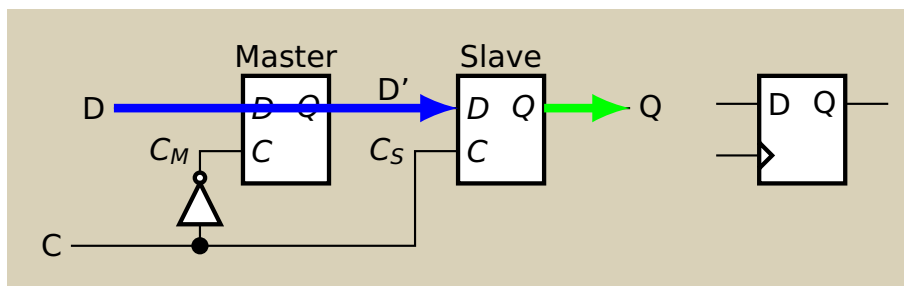
Positive-Edge-Triggered D Flip-Flop



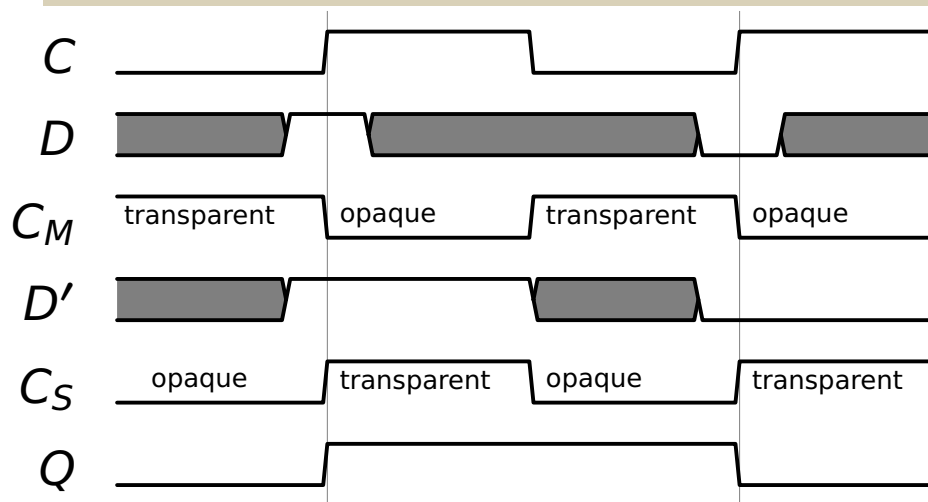
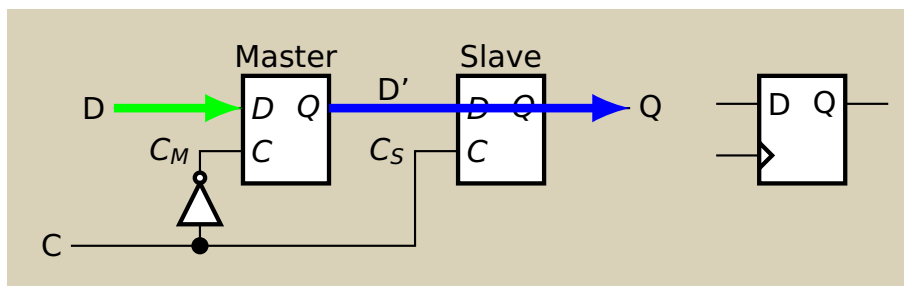
Positive-Edge-Triggered D Flip-Flop



Positive-Edge-Triggered D Flip-Flop

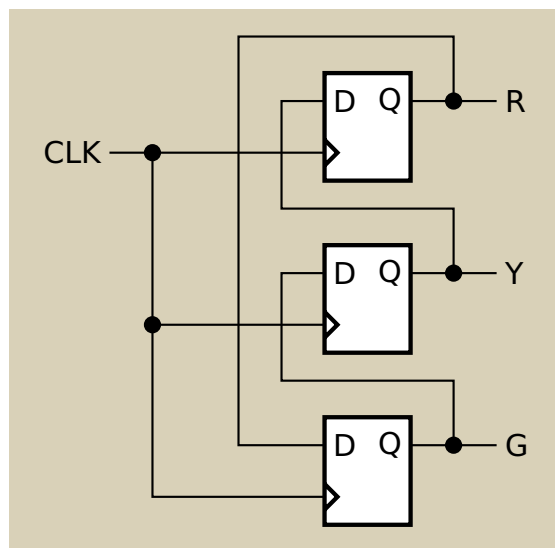


Positive-Edge-Triggered D Flip-Flop



The Traffic Light Controller: A second try

Let's try this again with D flip-flops.



CLK _____

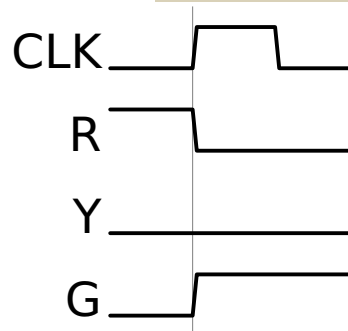
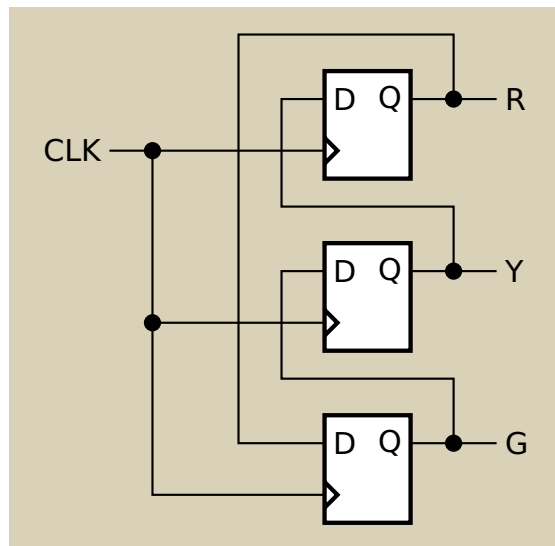
R _____

Y _____

G _____

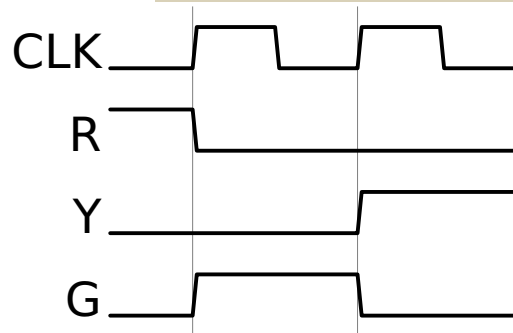
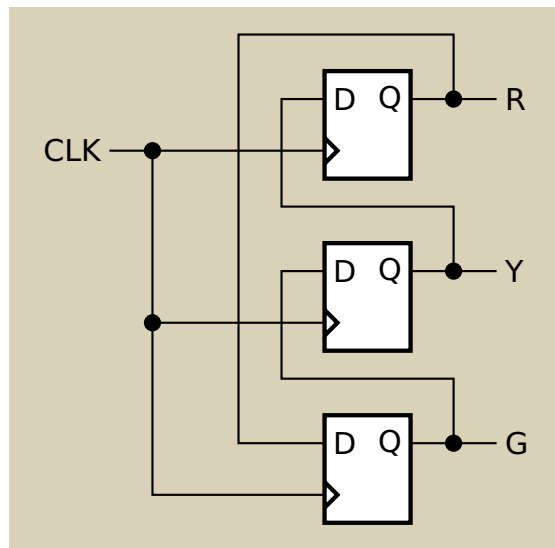
The Traffic Light Controller: A second try

Let's try this again with D flip-flops.



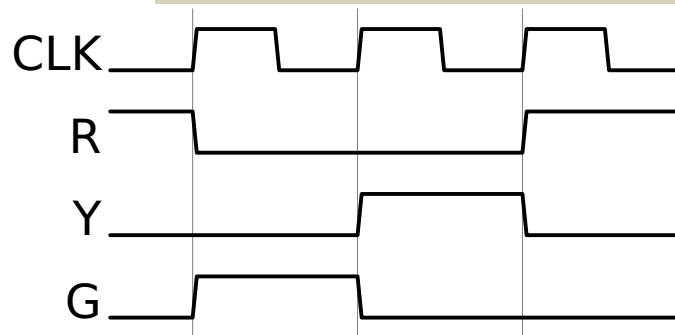
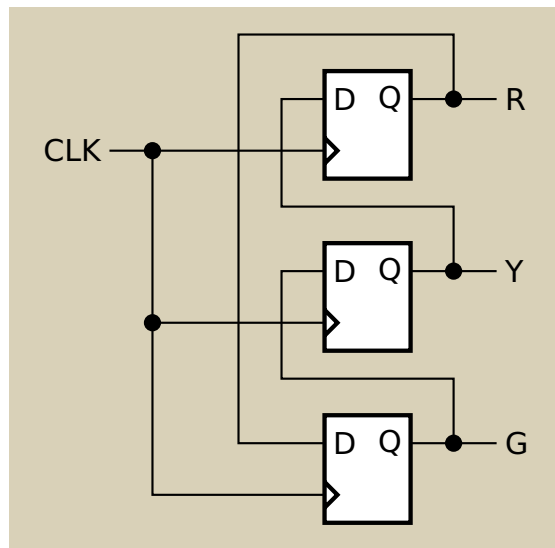
The Traffic Light Controller: A second try

Let's try this again with D flip-flops.



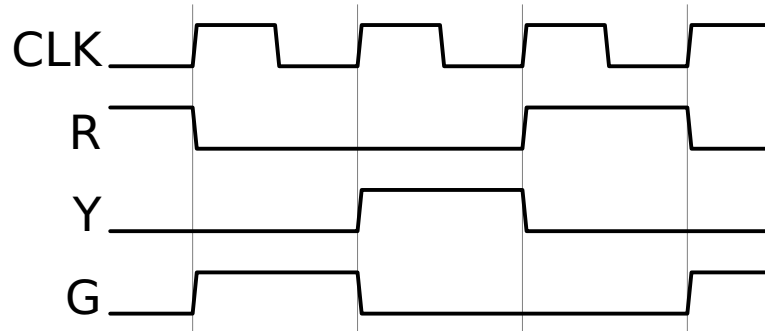
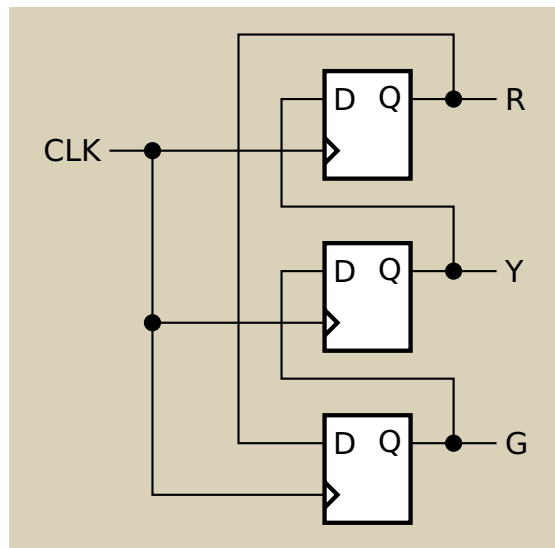
The Traffic Light Controller: A second try

Let's try this again with D flip-flops.

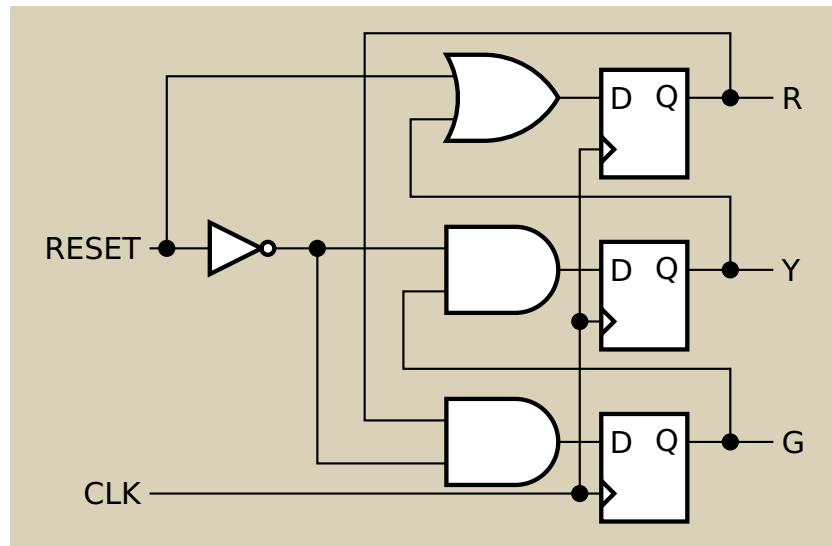


The Traffic Light Controller: A second try

Let's try this again with D flip-flops.

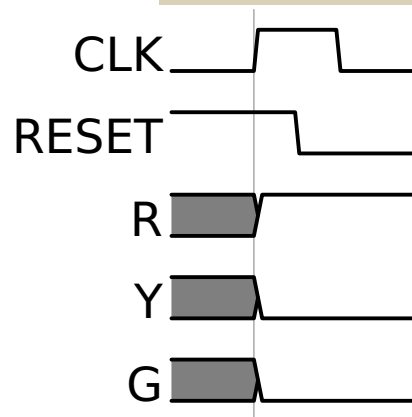
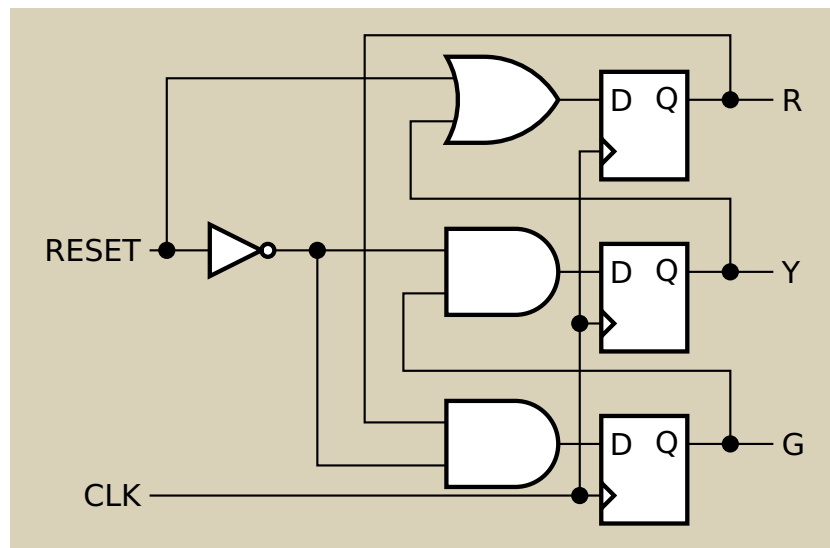


The Traffic Light Controller with Reset

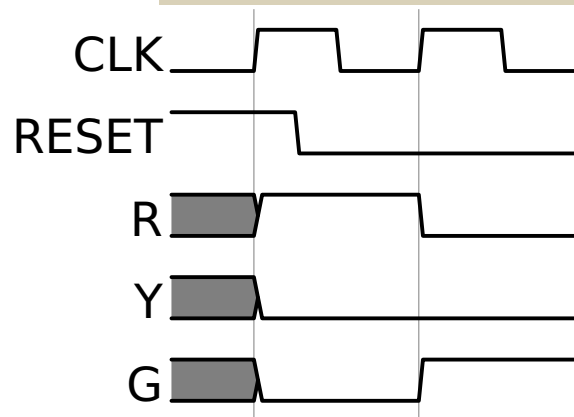
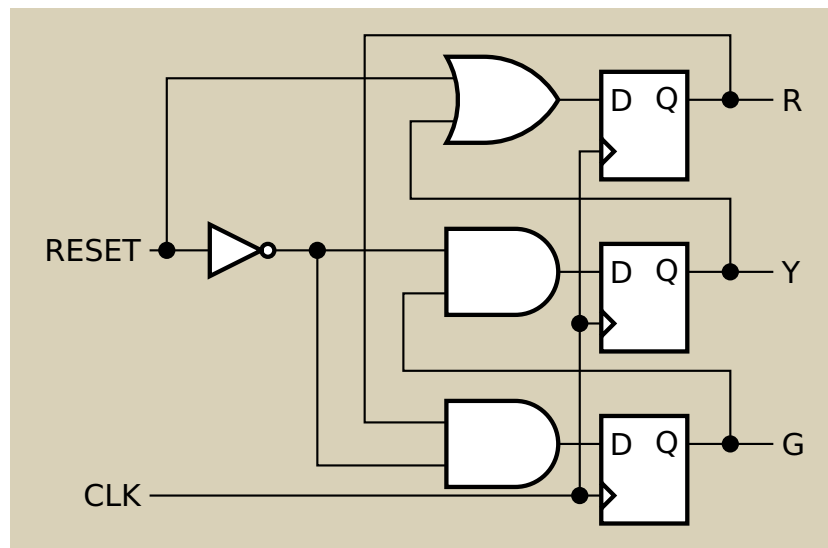


CLK _____
RESET _____
R
Y
G

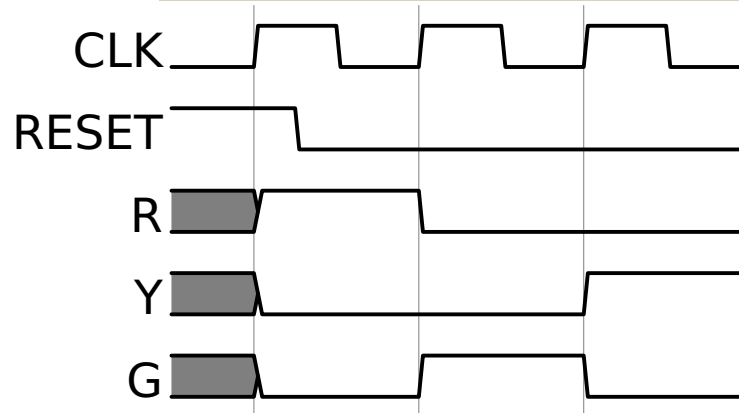
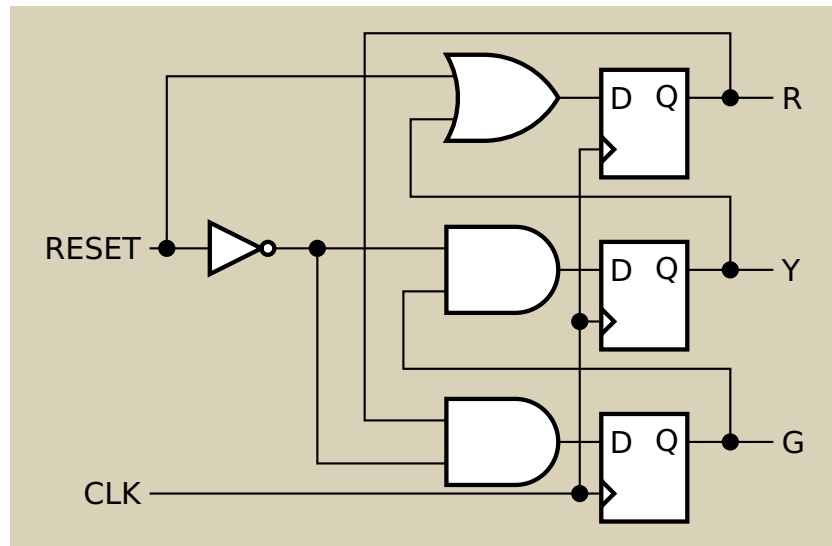
The Traffic Light Controller with Reset



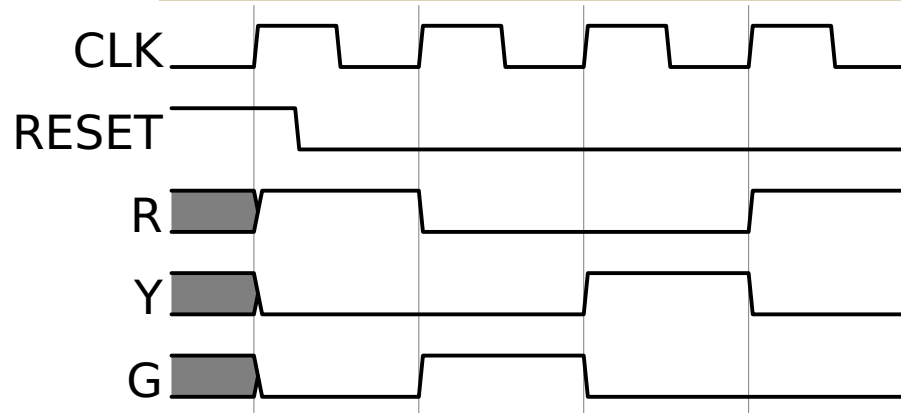
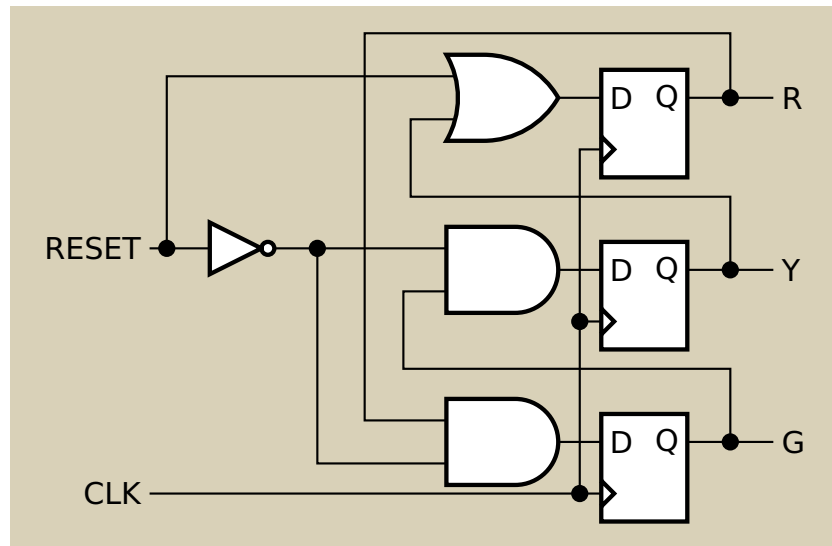
The Traffic Light Controller with Reset



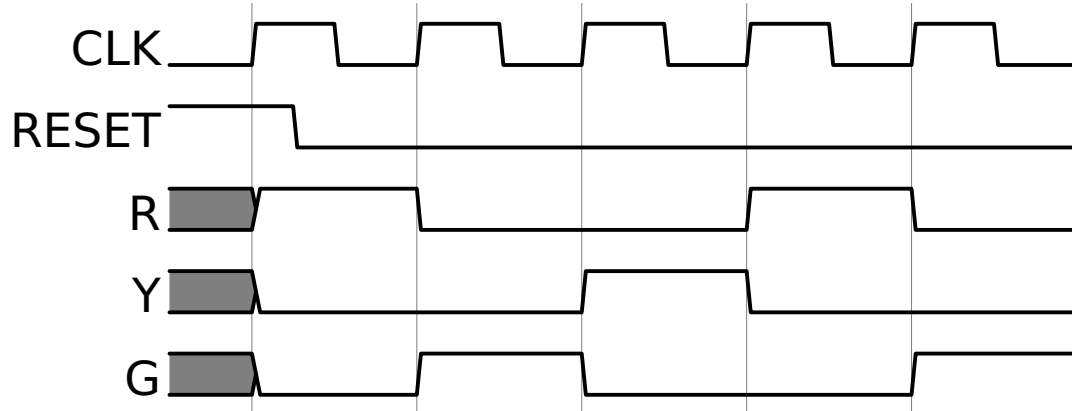
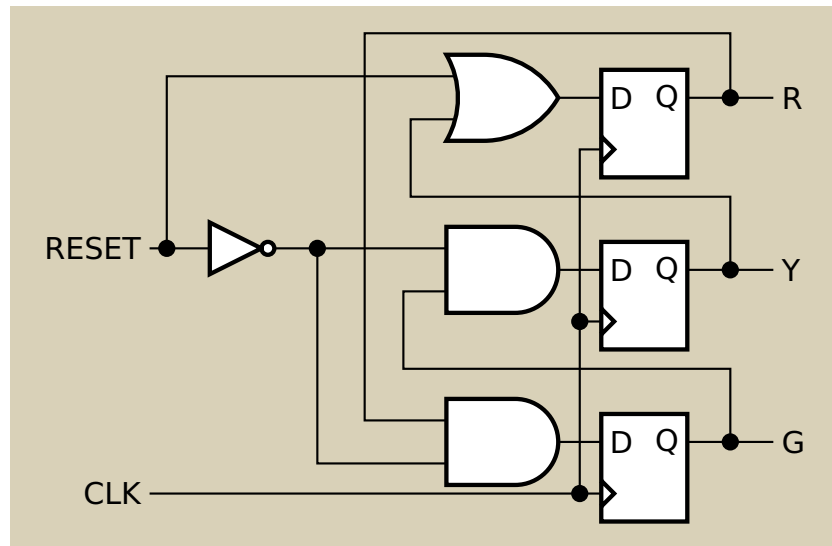
The Traffic Light Controller with Reset



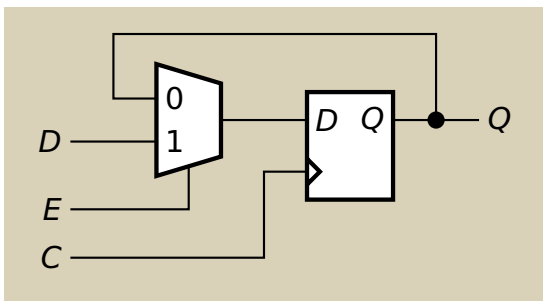
The Traffic Light Controller with Reset



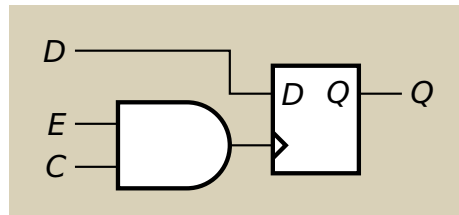
The Traffic Light Controller with Reset



D Flip-Flop with Enable

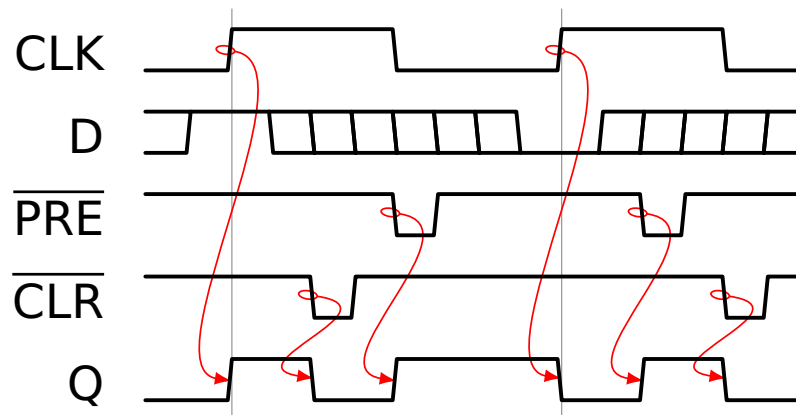
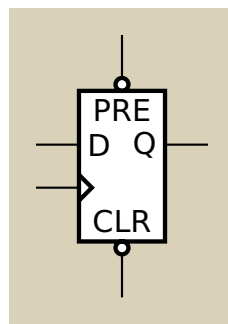


C	E	D	Q
↑	0	X	Q
↑	1	0	0
↑	1	1	1
0	X	X	Q
1	X	X	Q

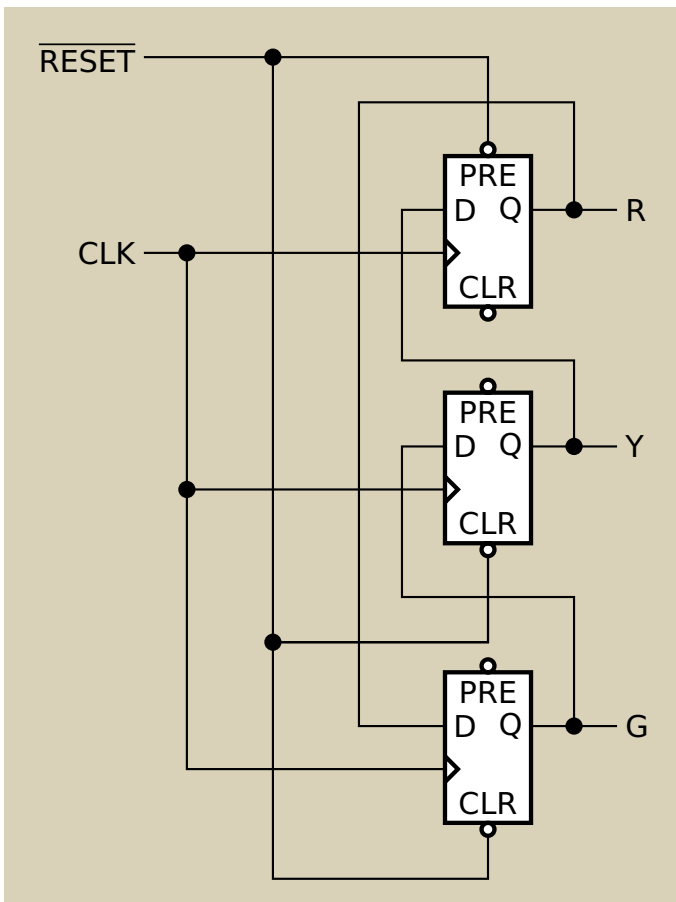


What's wrong with this solution?

Asynchronous Preset/Clear



The Traffic Light Controller w/ Async. Reset

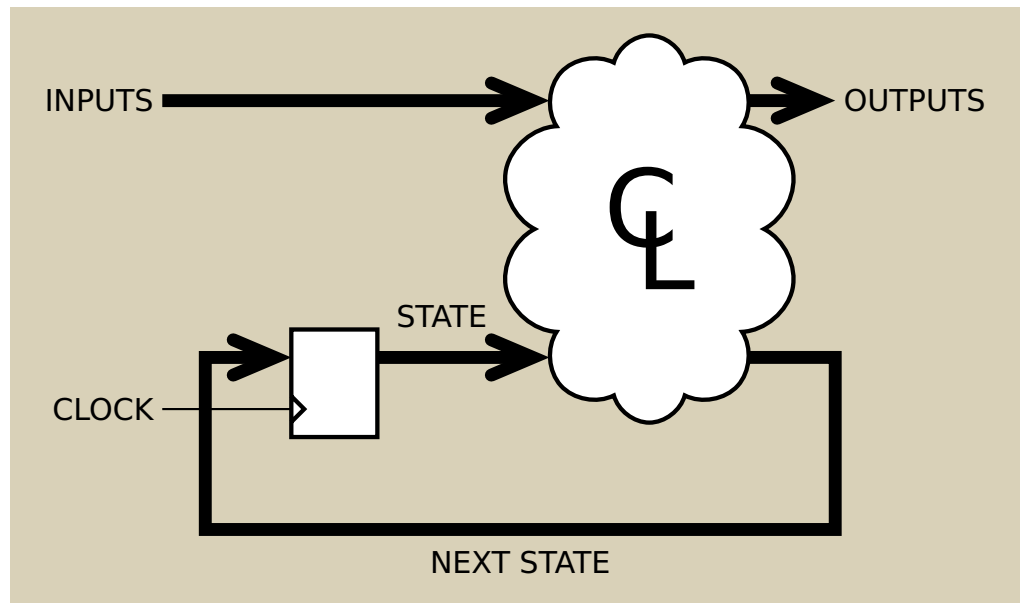


The Synchronous Digital Logic Paradigm

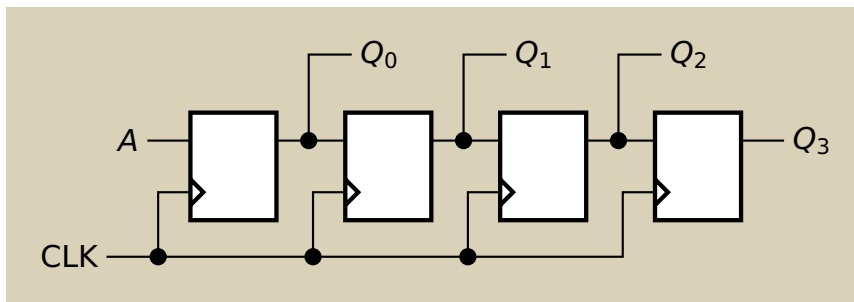
Gates and D flip-flops only

Each flip-flop driven by the same clock

Every cyclic path contains at least one flip-flop

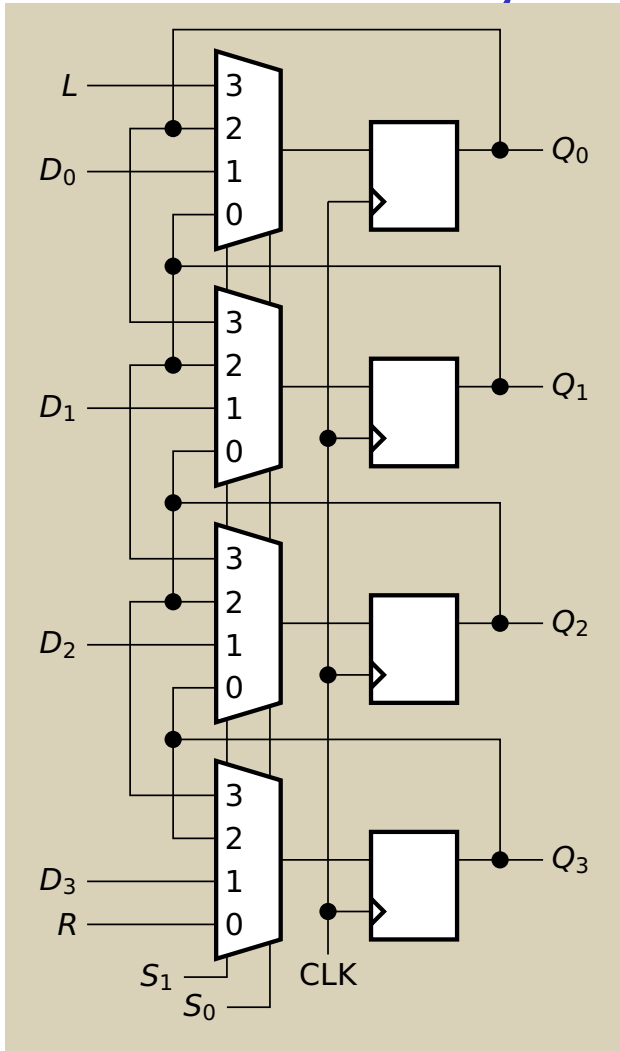


Cool Sequential Circuits: Shift Registers



A	Q ₀	Q ₁	Q ₂	Q ₃
0	X	X	X	X
1	0	X	X	X
1	1	0	X	X
0	1	1	0	X
1	0	1	1	0
0	1	0	1	1
0	0	1	0	1
0	0	0	1	0
1	0	0	0	1
0	1	0	0	0

Universal Shift Register

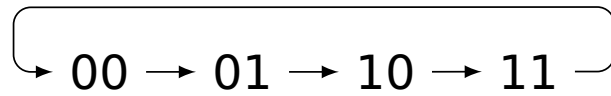


S_1	S_0	Q_3	Q_2	Q_1	Q_0
0	0	R	Q_3	Q_2	Q_1
0	1	D_3	D_2	D_1	D_0
1	0	Q_3	Q_2	Q_1	Q_0
1	1	Q_2	Q_1	Q_0	L

S_1	S_0	Operation
0	0	Shift right
0	1	Load
1	0	Hold
1	1	Shift left

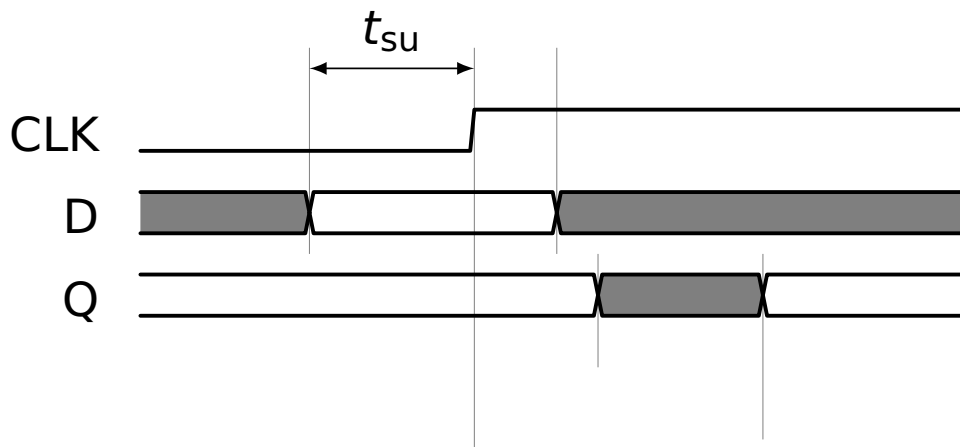
Cool Sequential Circuits: Counters

Cycle through sequences of numbers, e.g.,



Flip-Flop Timing

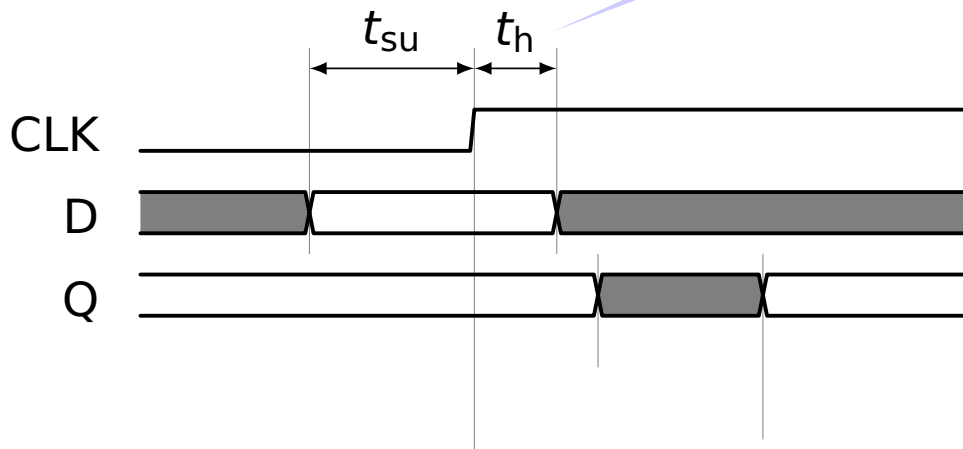
Setup Time: Time before the clock edge after which the data may not change



Flip-Flop Timing

Setup Time: Time before the clock edge after which the data may not change

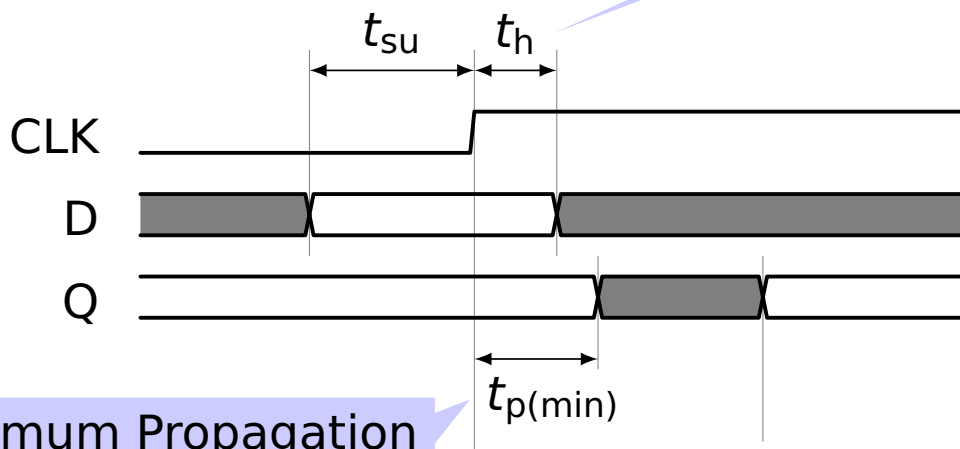
Hold Time: Time after the clock edge after which the data may change



Flip-Flop Timing

Setup Time: Time before the clock edge after which the data may not change

Hold Time: Time after the clock edge after which the data may change

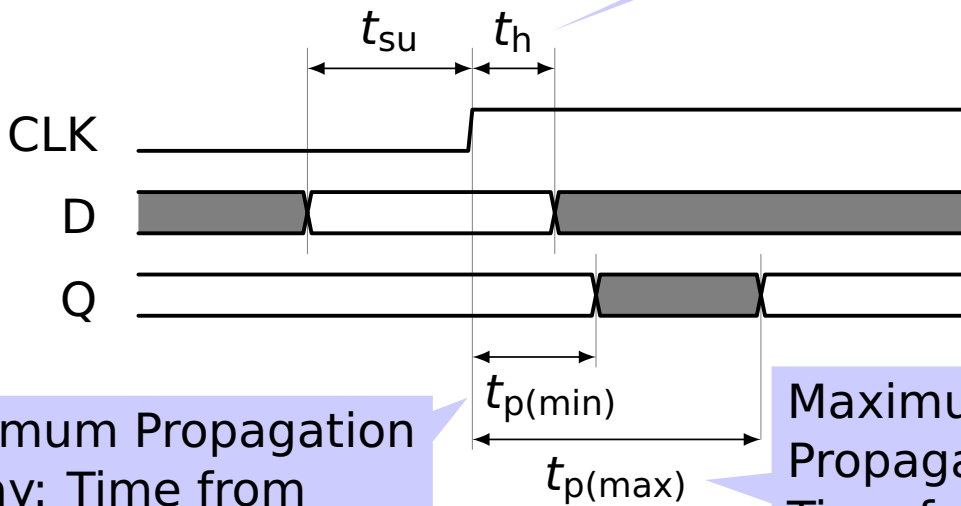


Minimum Propagation Delay: Time from clock edge to when Q might start changing

Flip-Flop Timing

Setup Time: Time before the clock edge after which the data may not change

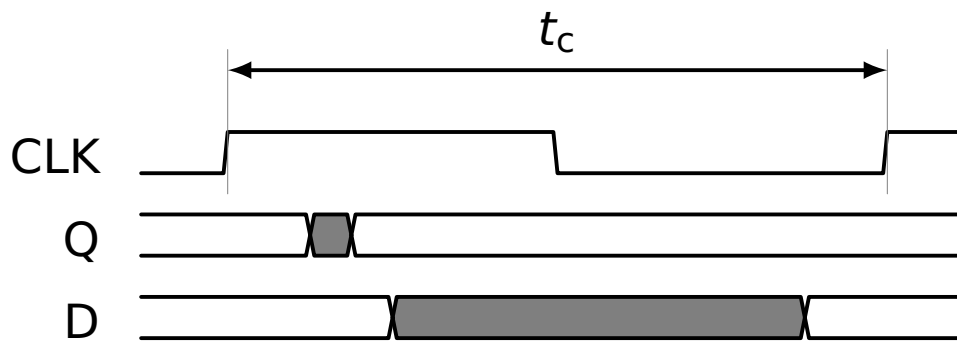
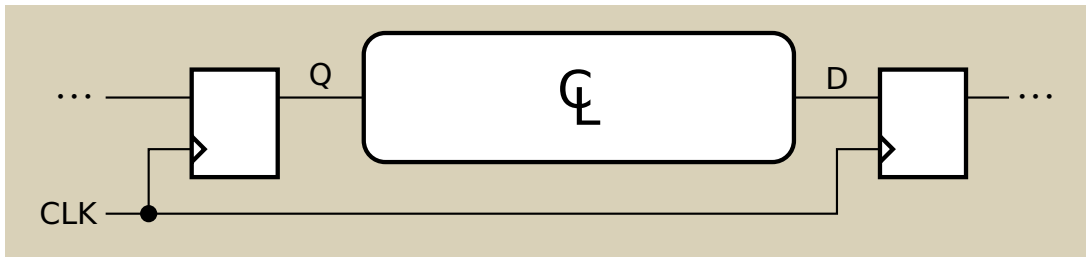
Hold Time: Time after the clock edge after which the data may change



Minimum Propagation Delay: Time from clock edge to when Q might start changing

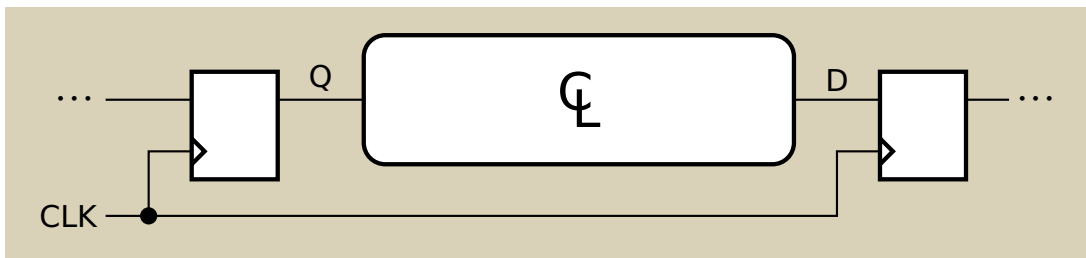
Maximum Propagation Delay: Time from clock edge to when Q guaranteed stable

Timing in Synchronous Circuits

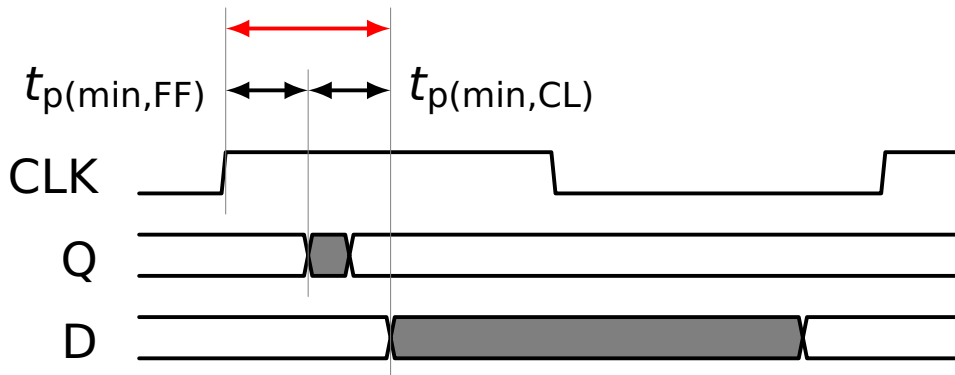


t_c : Clock period. E.g., 10 ns for a 100 MHz clock

Timing in Synchronous Circuits

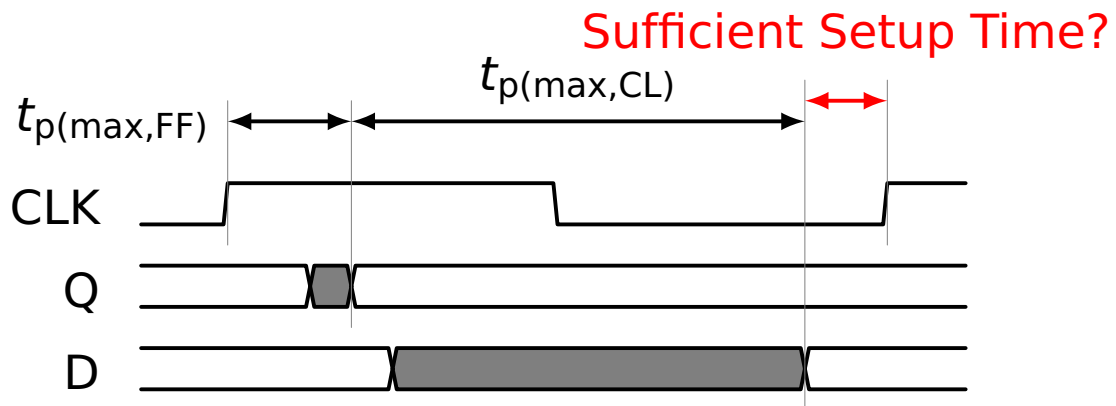
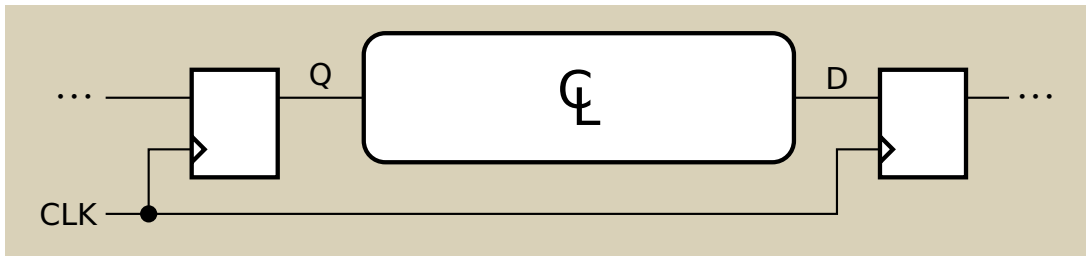


Sufficient Hold Time?



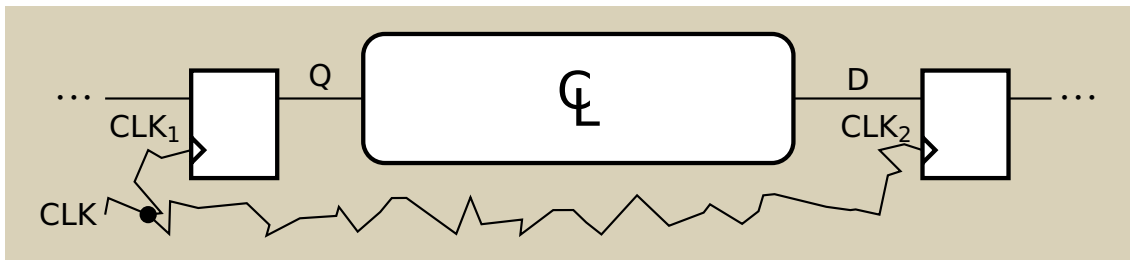
Hold time constraint: how soon after the clock edge is D liable to start changing?
Min. FF delay + min. logic delay

Timing in Synchronous Circuits

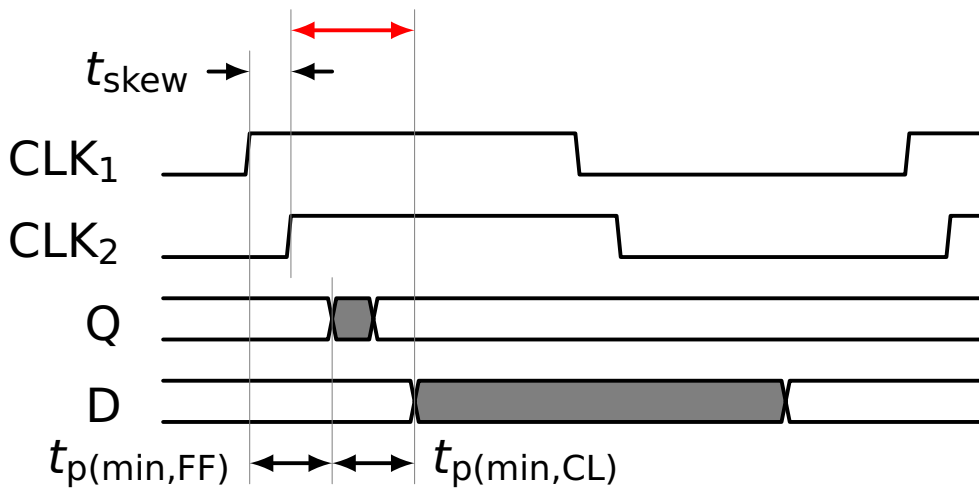


Setup time constraint: when before the clock edge is D guaranteed to have stabilized?
Max. FF delay + max. logic delay

Clock Skew: What Really Happens

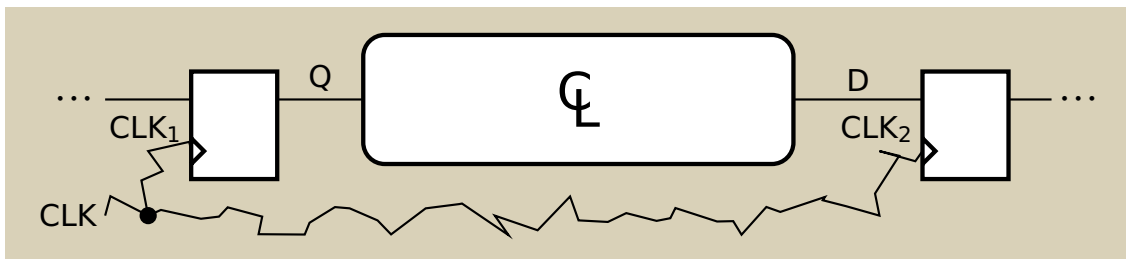


Sufficient Hold Time?

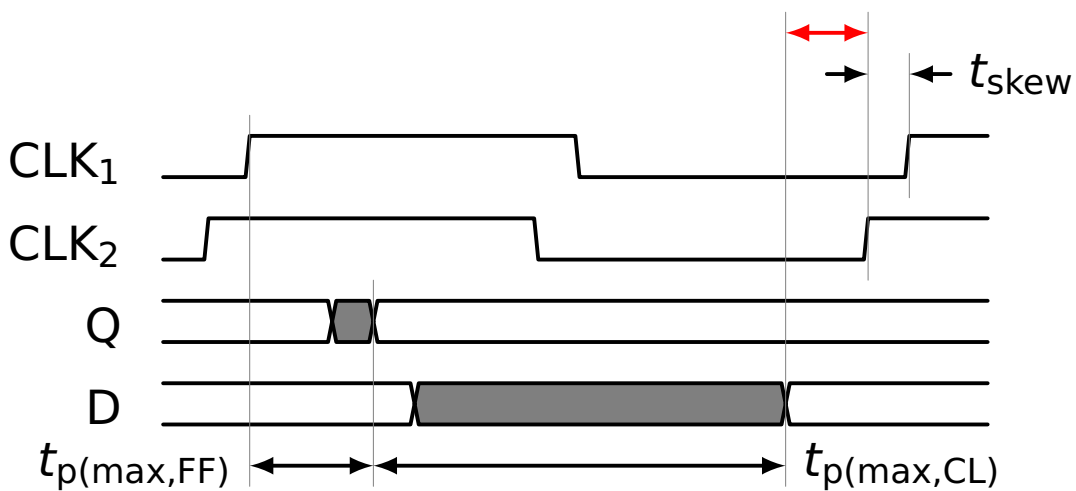


CLK₂ arrives late, creating potential hold time violation

Clock Skew: What Really Happens



Sufficient Setup Time?



CLK_2 arrives early, creating potential setup time violation