## Algorithmic Complexity

Prof. Rebecca N. Wright

NOTE: This homework assignment is to be completed in pairs. Each pair should work together and turn in a single solution. Both pair members are expected to contribute to finding the solution and to be able to explain everything written in the solution.

If at all possible, please use an appropriate text-processing package to write up your solutions, such as LaTeX or MS-Word. If necessary, you may leave space for mathematical formulas and drawings and insert them by hand. Provide justification for all answers.

Each problem is worth 15 points.

Problem 1: Show that the following problem, Paths With Forbidden Pairs is NP-complete.
Instance: A directed graph $G=(V, E)$, specified vertices $s, t \in V$, and a collection $C=\left\{\left(a_{1}, b_{1}\right),\left(a_{2}, b_{2}\right), \ldots,\left(a_{n}, b_{n}\right)\right\}$ of pairs of vertices from $V$.
Question: Is there a directed path from $s$ to $t$ in $G$ that contains at most one vertex from each pair in $C$ ?

Problem 2: Show that the Independent Set problem for bipartite graphs can be solved in polynomial time. That is, show a polynomial time for determining if a given bipartite graph has an independent set of at least a given size $J$ ?

