

Landscape Generator Based on FFT 3D-Spectral Visualization

- CSEE 4840 Group FFT

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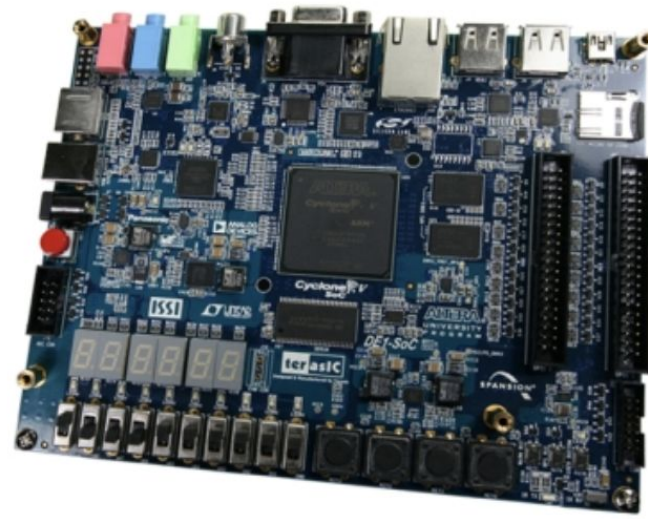
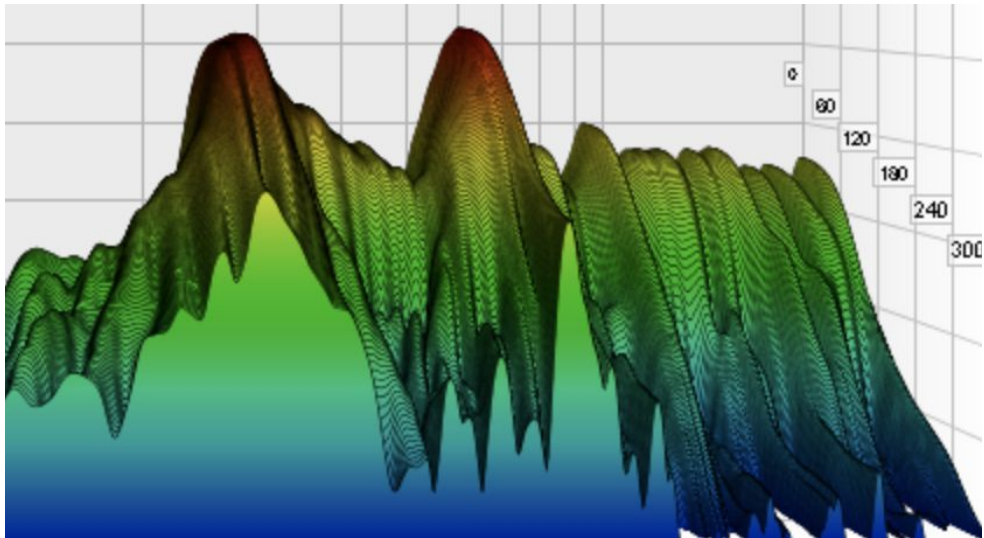
Demo

Overview

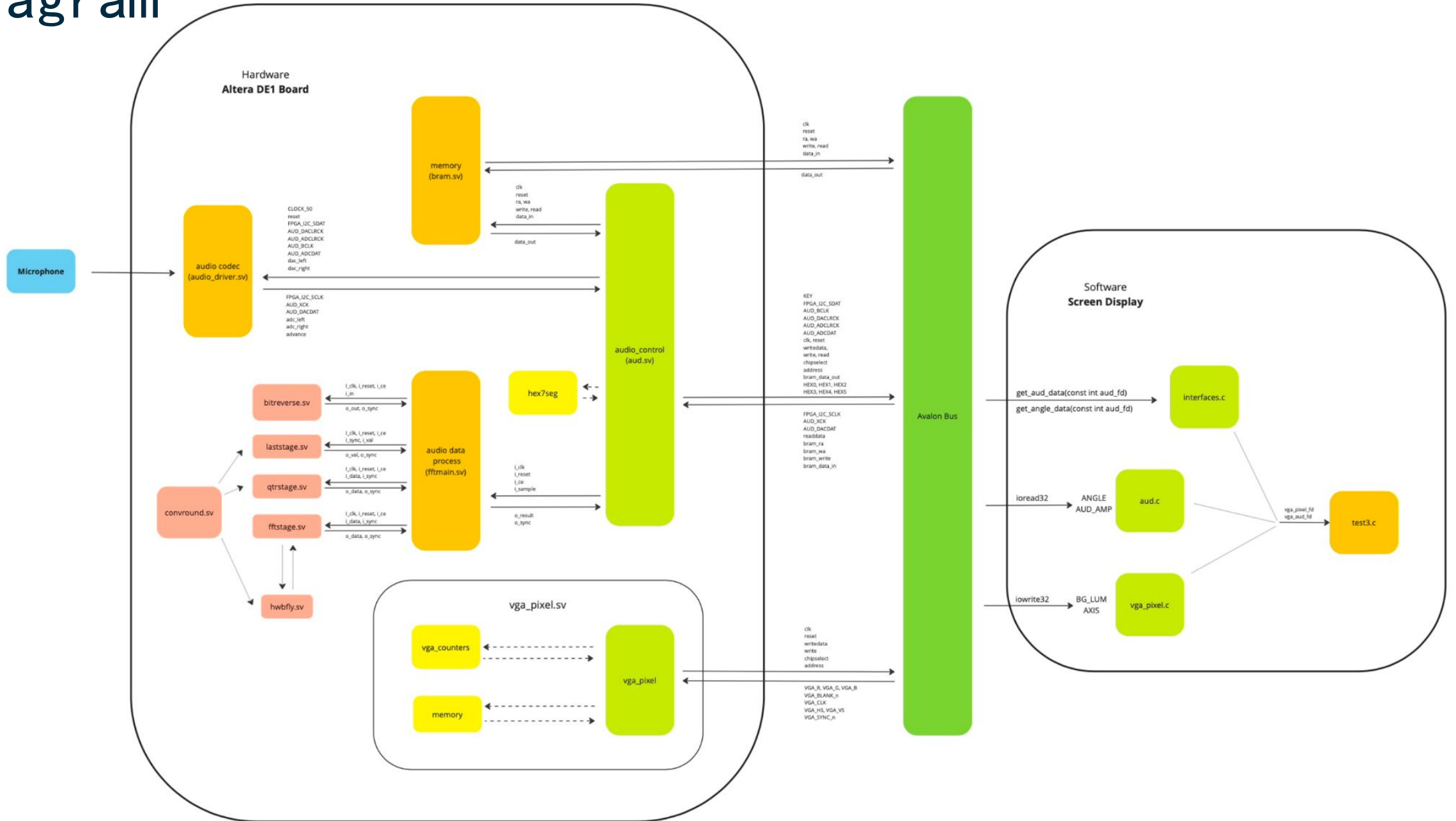
- Overview
- Diagram

Overview

- What we have...
 1. Audio Data Processing – Obtain audio from microphone
 2. FFT (Fast Fourier transform) Calculation – Execute FFT on audio data
 3. Data Visualization – Show FFT Results on VGA as dynamic waterfall waveform
 4. Button Control – Adjust the shape of the visualization (30 – 60 degree)



Diagram

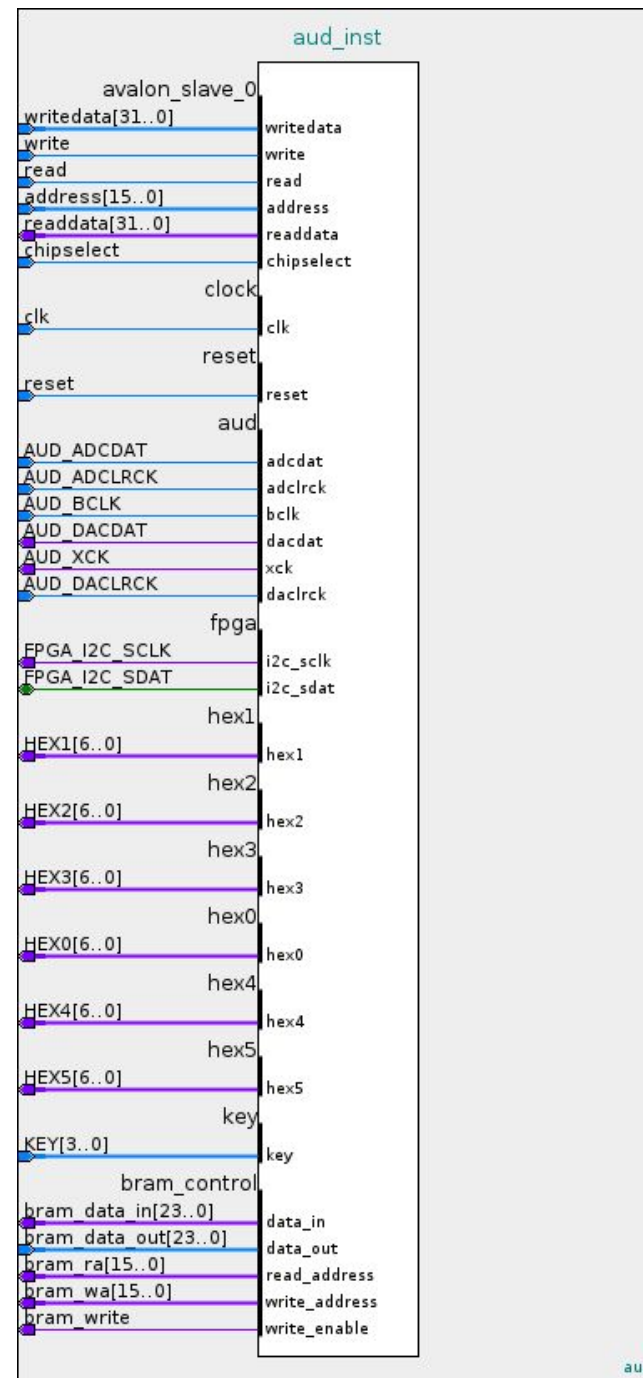


Hardware

- Audio
- DIF FFT
- VGA Display

Hardware – Audio

The Audio module synthesizes most of the functions, including sound acquisition, FFT, memory management, key control, and Hex7seg.



Hardware – FFT Module

- DIF FFT**

$$\begin{aligned}
 X[2k] &= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_{\frac{N}{2}}^{kn} + \sum_{n=\frac{N}{2}}^{\frac{N}{2}-1} x\left[n + \frac{N}{2}\right] W_{\frac{N}{2}}^{k\left(n+\frac{N}{2}\right)} \\
 &= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_{\frac{N}{2}}^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n + \frac{N}{2}\right] W_{\frac{N}{2}}^{kn} \\
 &= DFT_{\frac{N}{2}} \left\{ x[n] + x\left[n + \frac{N}{2}\right] \right\}
 \end{aligned}$$

$$X[2k + 1] = DFT_{\frac{N}{2}} \left\{ W_N^n (x[n] - x\left[n + \frac{N}{2}\right]) \right\}$$

- Difference between DIF and DIT FFT**

- The output is bit-reversed
- Multiplication is done after butterfly

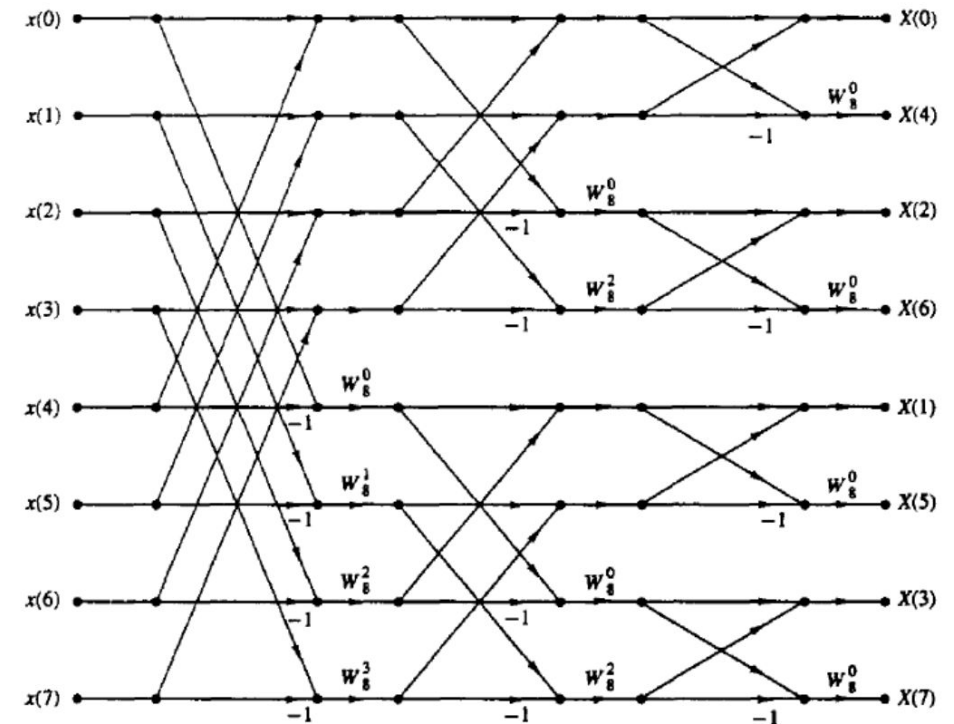
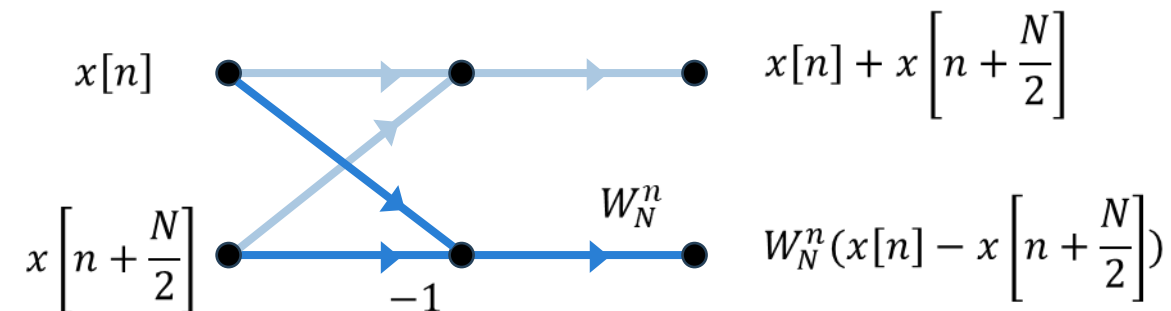


Figure: N=8 point decimation in frequency FFT algorithm

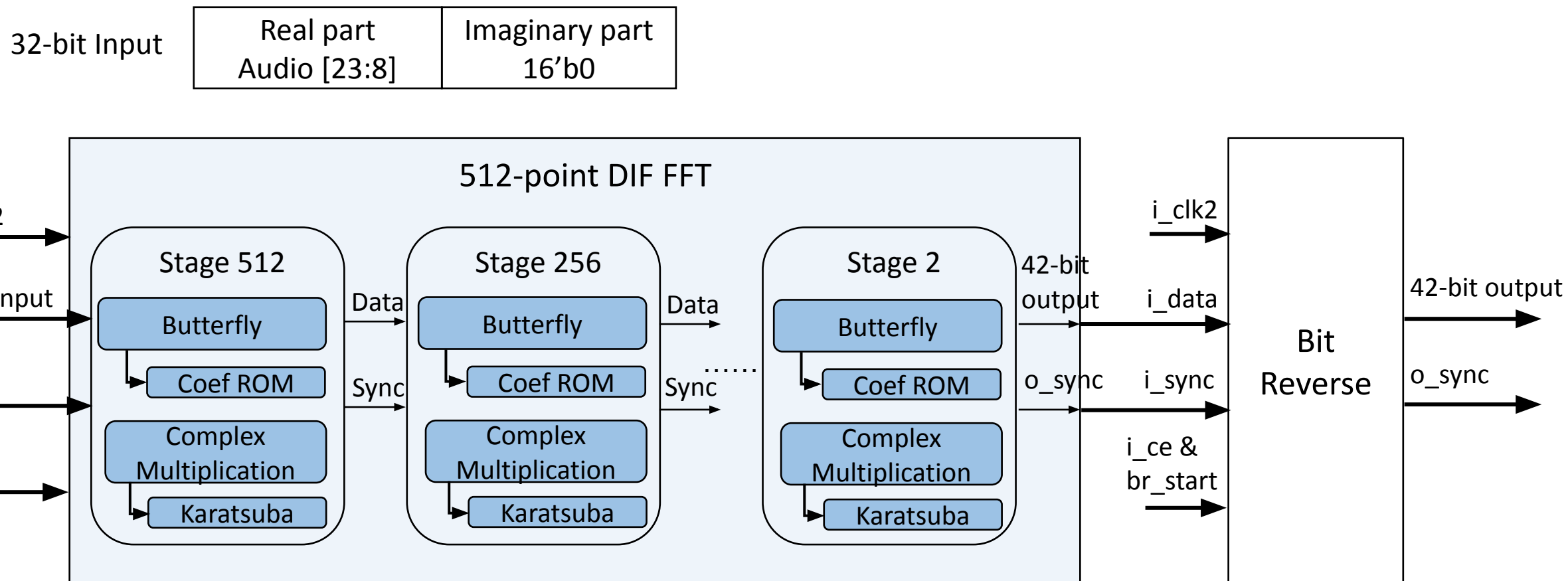


[1] THE FAST FOURIER TRANSFORM (FFT). (n.d.-a). <https://eeweb.engineering.nyu.edu/iselesni/EL713/zoom/fft>

[2] Proakis, J. G., & Manolakis, D. G. (1996). Digital Signal Processing: Principles, algorithms, and applications. Prentice Hall.

Hardware - FFT Module

- FFT Module Architecture



Hardware - FFT Module

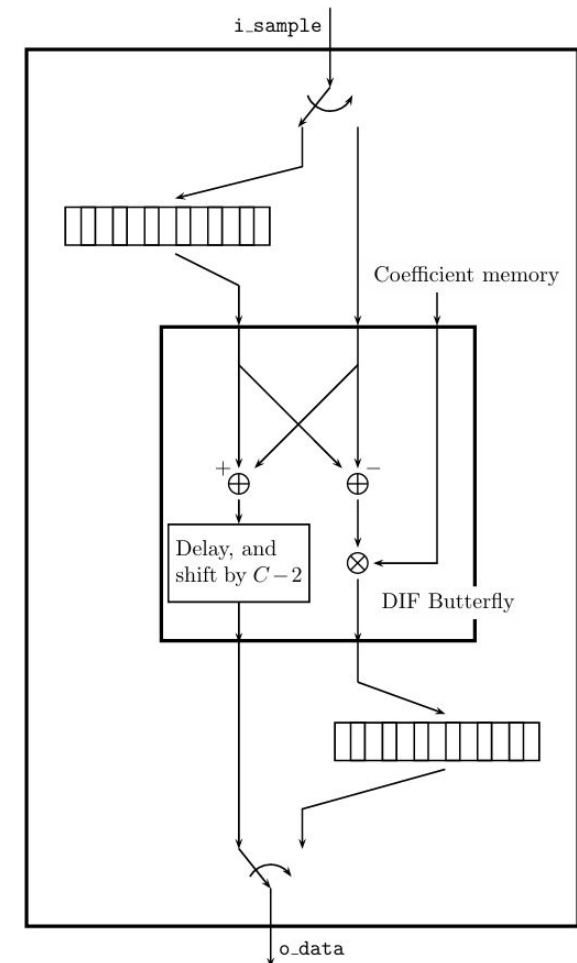
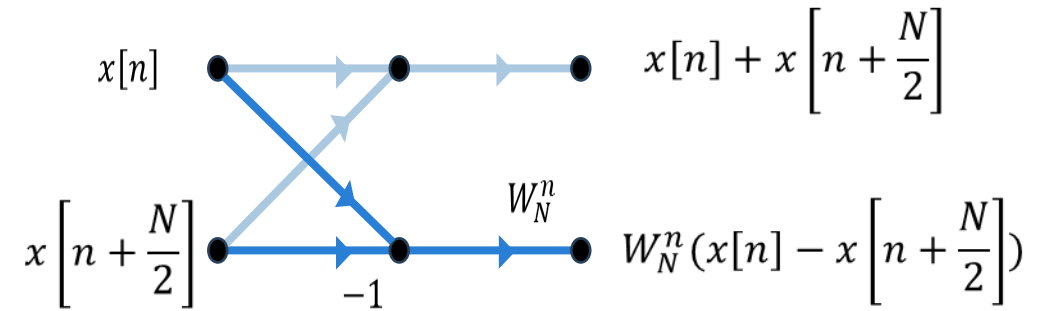
- Internal Stage Architecture

- Internal Calculation

- Butterfly Operation
- Complex Multiplication

- Butterfly - Add and Subtraction

- $O_1 = A + B$
- $O_2 = (A - B) * C$



Hardware - FFT Module

- Internal Stage Architecture

3. Coefficient - Twiddle Factor $W_N^k = e^{-j\frac{2\pi k}{N}}$

- Coefficients for Different Stages are Pre-Stored in

Memory $W_N^{N-k} = -W_N^k$

- The upper bits are the real part of the coefficient;
- The lower bits are the imaginary part of the coefficient.

cmem_16.hex (N = 16, k : 0 ~ 7)

```
10 // Each line contains a coefficient. The real portion
11 // of the coefficient is in the upper 23 bits, whereas
12 // the lower 23 bits contain the imaginary portion
13 //
14 //
15 100000000000
16 0ec83673c10f
17 0b504f695f62
18 061f78e26f94
19 000000600000
20 39e087e26f94
21 34afb1695f62
22 3137ca73c10f
```

Hardware - FFT Module

- Fast Multiplication Algorithm - Karatsuba Algorithm

1. Basic Idea

$$\begin{cases} M = m_1 + jm_2 \\ N = n_1 + jn_2 \end{cases}$$

$$M \times N = \begin{matrix} \downarrow \\ m_1 \cdot n_1 - m_2 \cdot n_2 \\ + m_1 \cdot n_2j + m_2 \cdot n_1j \end{matrix}$$

4 Multiplications

3 Multiplications

$$\begin{cases} P1 = m_1 \cdot n_1 \\ P2 = m_2 \cdot n_2 \\ P3 = (m_1 + m_2) \cdot (n_1 + n_2) \end{cases}$$

$$M \times N = (P1 - P2) + j(P3 - P1 - P2)$$

2. Implementation in Butterfly Operation in DIF FFT

$$\begin{matrix} A = a_1 + ja_2 \\ B = b_1 + jb_2 \\ C = c_1 + jc_2 \end{matrix} \longrightarrow \begin{matrix} O_1 = A + B \\ O_2 = (A - B) * C \end{matrix}$$

$$\begin{cases} P1 = (a_1 - b_1) \cdot c_1 \\ P2 = (a_2 - b_2) \cdot c_2 \\ P3 = [(a_1 - b_1) + (a_2 - b_2)] \cdot (c_1 + c_2) \end{cases}$$

$$\longrightarrow O_2 = (P1 - P2) + j(P3 - P1 - P2)$$

Hardware - FFT Module

- Fast Multiplication Algorithm - Karatsuba Algorithm

$$\begin{aligned} A &= a_1 + ja_2 \\ B &= b_1 + jb_2 \\ C &= c_1 + jc_2 \end{aligned}$$

$$\begin{aligned} O_1 &= A + B \\ O_2 &= (A - B) * C \end{aligned}$$

$$\left[\begin{aligned} P1 &= (a_1 - b_1) \cdot c_1 \\ P2 &= (a_2 - b_2) \cdot c_2 \\ P3 &= [(a_1 - b_1) + (a_2 - b_2)] \cdot (c_1 + c_2) \end{aligned} \right.$$

```
always @(posedge i_clk)
if (i_ce)
begin
// One clock just latches the inputs
r_left <= i_left; // No change in # of bits
r_right <= i_right;
r_coef <= i_coef;
// Next clock adds/subtracts
r_sum_r <= r_left_r + r_right_r; // Now IWIDTH+1 bits
r_sum_i <= r_left_i + r_right_i;
r_dif_r <= r_left_r - r_right_r;
r_dif_i <= r_left_i - r_right_i;
// Other inputs are simply delayed on second clock
ir_coef_r <= r_coef[(2*CWIDTH-1):CWIDTH];
ir_coef_i <= r_coef[(CWIDTH-1):0];
end
```

```
always @(posedge i_clk)
if (i_ce)
begin
// Second clock, pipeline = 1
p1c_in <= ir_coef_r; //c1
p2c_in <= ir_coef_i; //c2
p1d_in <= r_dif_r; //a1-b1
p2d_in <= r_dif_i; //a2-b2
p3c_in <= ir_coef_i + ir_coef_r; // c1+c2
p3d_in <= r_dif_r + r_dif_i; //a1-b1+a2-b2
end
// }}}

// Perform our multiplies

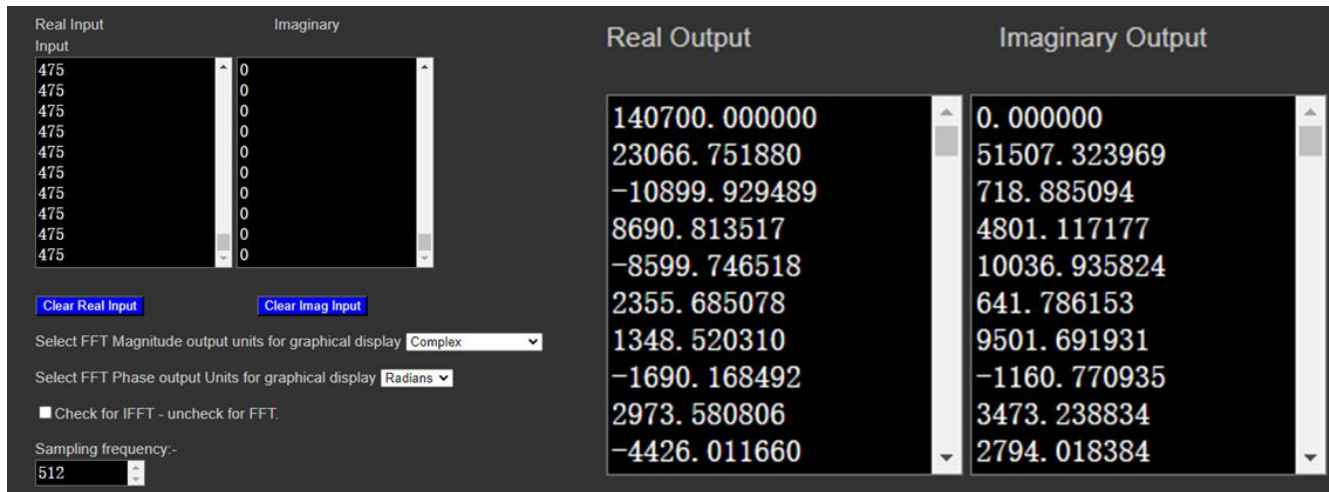
always @(posedge i_clk)
if (i_ce)
begin
// Third clock, pipeline = 3
// As desired, each of these lines infers a DSP48
rp_one <= p1c_in * p1d_in;
rp_two <= p2c_in * p2d_in;
rp_three <= p3c_in * p3d_in;
end
```

$$(A - B) * C$$

Hardware – FFT Module

- FFT Result Checking

Mathematical FFT Output

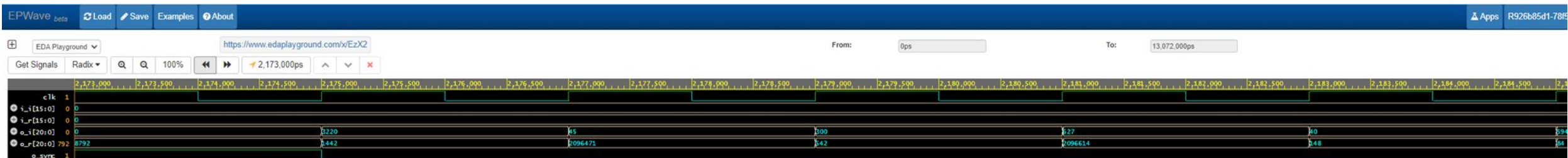


Testbench – Simulation of audio input data

```
//Test stimulus
initial begin

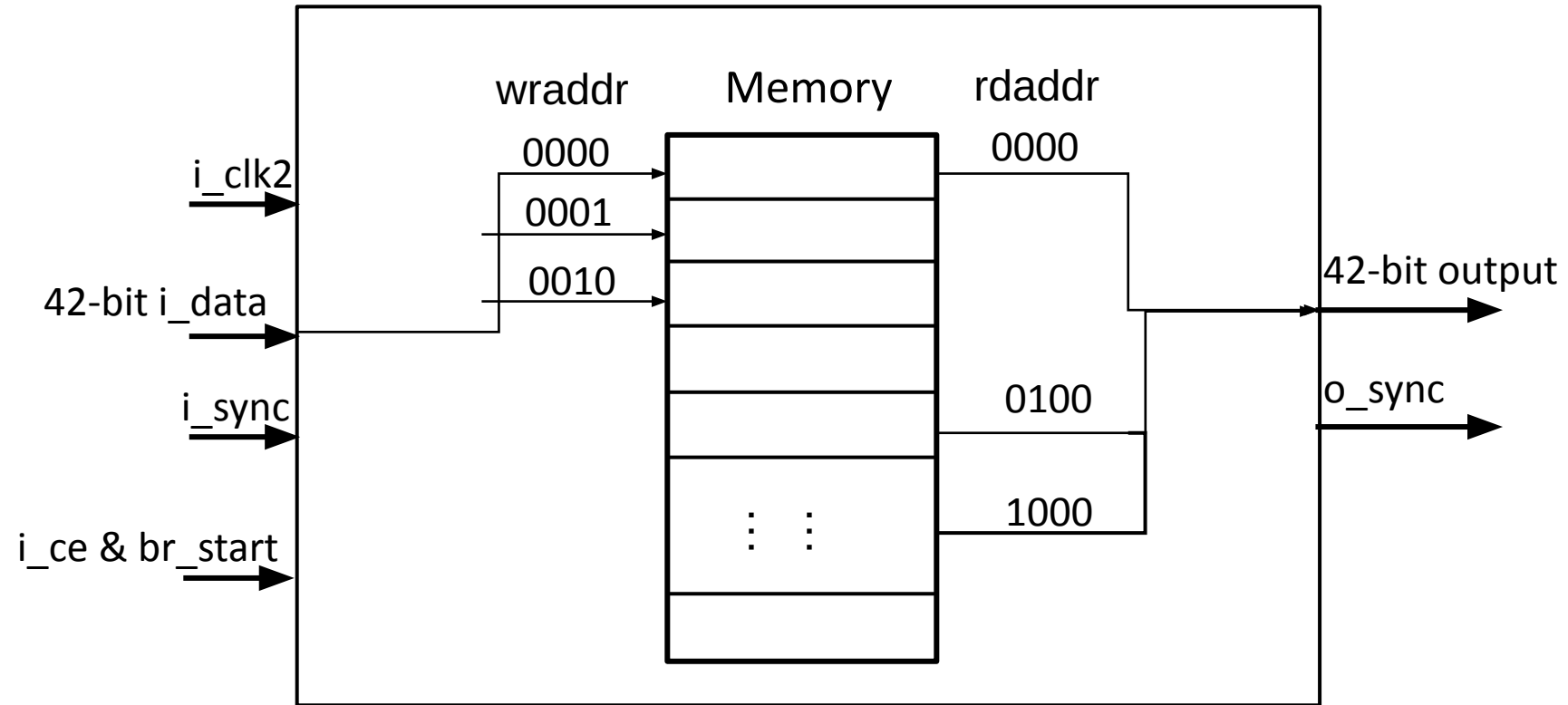
    // Test Case 1
    i_sample = {16'd200, 16'd0}; #200
    i_sample = {16'd100, 16'd0}; #400
    i_sample = {16'd475, 16'd0}; #424
```

Output Waveform



Hardware - FFT Module

- Bit Reverse Module



- Memory size: $2^9=512$ 42-bit data
- The `rdaddr` is computed as the bit-reversal of `wraddr`
- `o_sync` is set 1 when the bit-reversal process is done and the data output is valid.

Hardware – FFT Module

- Square Root Calculator Function

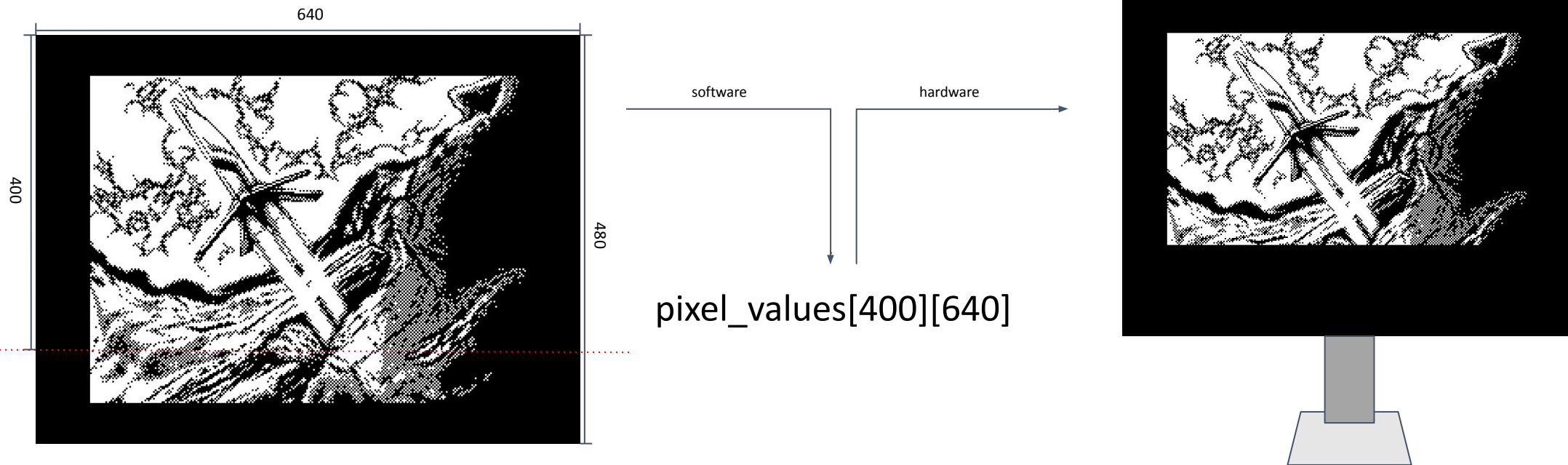
Step	A	X	T	Q	Description
	00000000	01111001	00000000	0000	Starting values.
1	00000001	11100100	00000000	0000	Left shift X by two places into A. Set T = A - {Q,01}: 01 - 01.
	00000000			0001	Left shift Q. Is T ≥ 0? Yes. Set A=T and Q[0]=1.
2	00000011	10010000	11111100	0010	Left shift X by two places into A. Set T = A - {Q,01}: 11 - 101.
					Left shift Q. Is T ≥ 0? No. Move to next step.
3	00001110	01000000	00000101	0100	Left shift X by two places into A. Set T = A - {Q,01}: 1110 - 1001
	00000101			0101	Left shift Q. Is T ≥ 0? Yes. Set A=T and Q[0]=1.
4	00010101	00000000	00000000	1010	Left shift X by two places into A. Set T = A - {Q,01}: 10101 - 10101.
	00000000			1011	Left shift Q. Is T ≥ 0? Yes. Set A=T and Q[0]=1.

1. Left shift the largest 2 bit of number and add with remain.
2. Compare the value to {answer,01}
3. Update lowest bit of answer and remain value.

Hardware – VGA

- Design

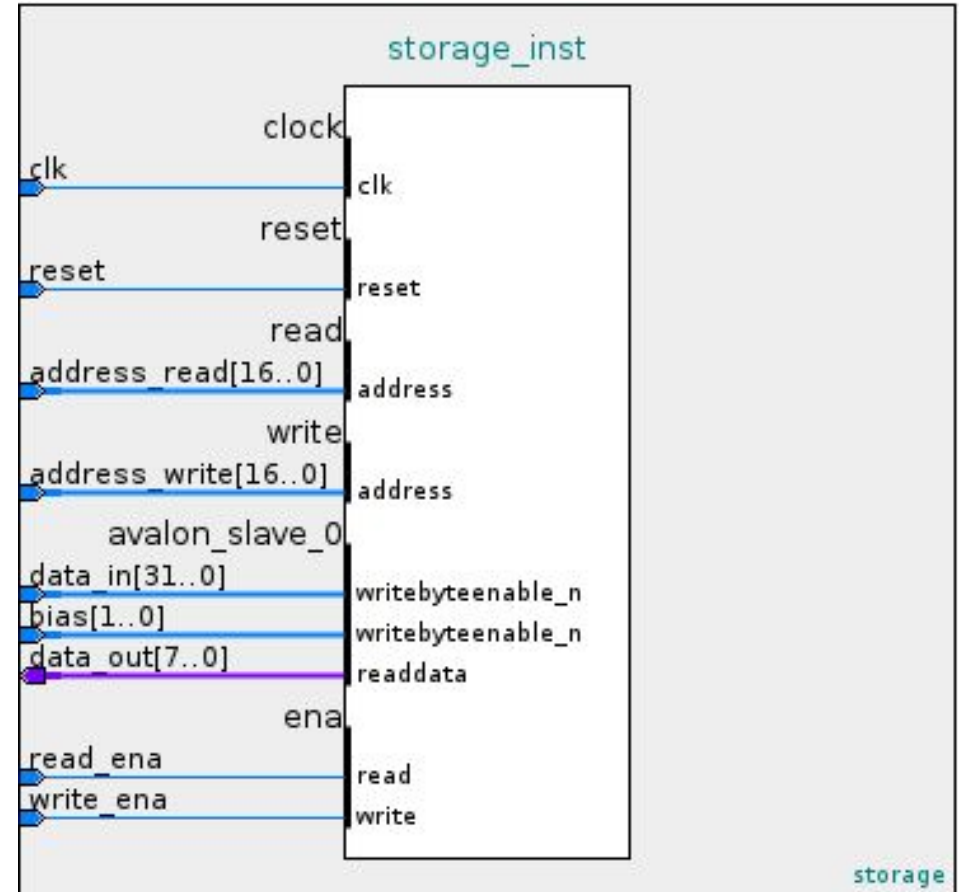
- 256000(640 * 400) bit grayscale output

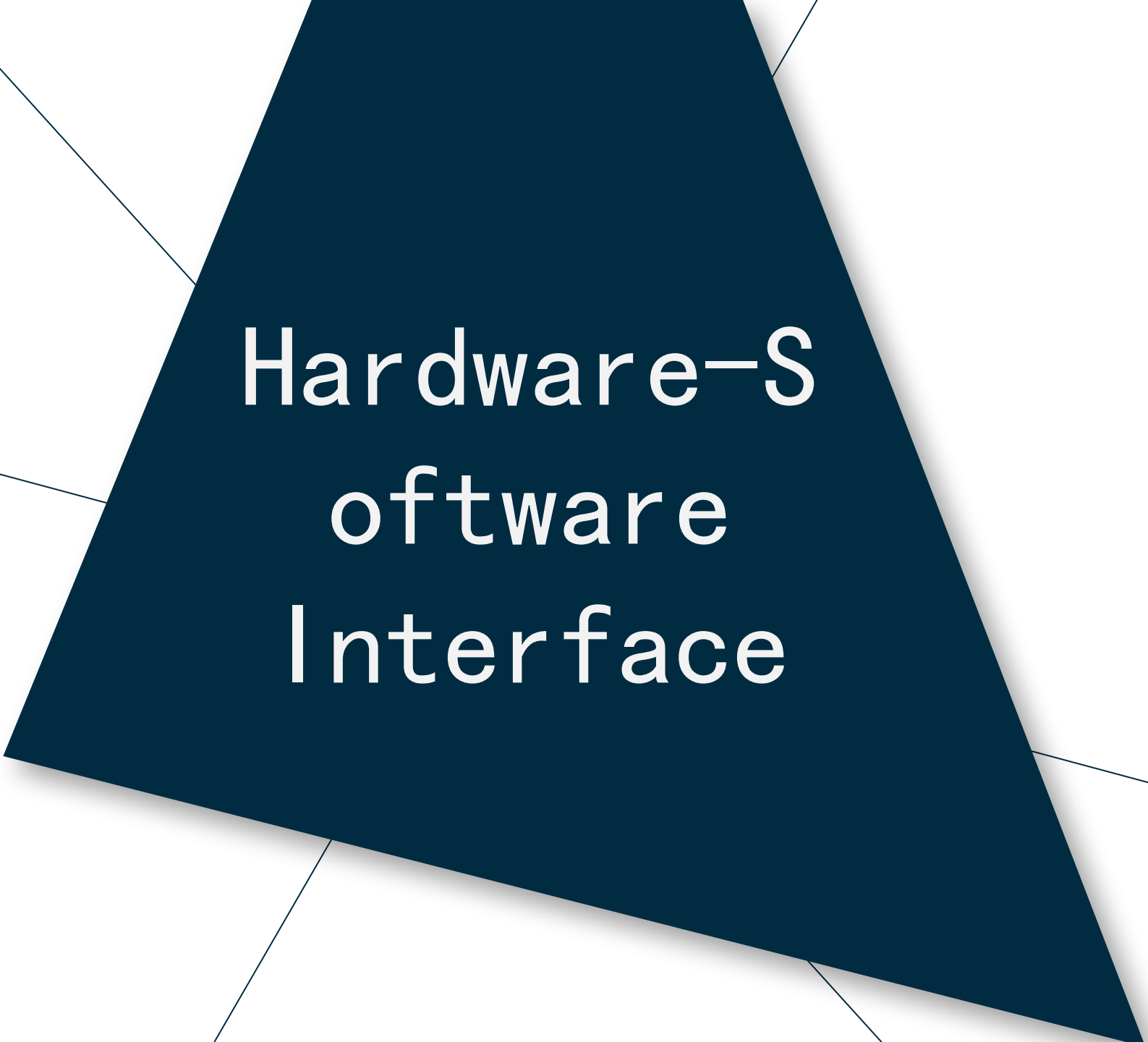


Hardware – VGA

- **Frame Storage**

- Use memory to store the frame
- Read memory through VGA counter cycle



A dark blue triangle pointing downwards, centered on the page. It has a slight drop shadow and is surrounded by thin, light blue lines that form a grid-like pattern.

Hardware-Software Interface



Software

Software

- **Data Capture**

- Audio Data Acquisition

- `get_aud_data(aud_fd)`

- Angle Data Acquisition

- `get_angle_data(aud_fd)`



- **Data Processing and Waveform Generation**

- Frames and CUR Arrays

- Frames: Stores historical audio data over multiple frames.

- CUR: Holds the current state of the waveform to be displayed.

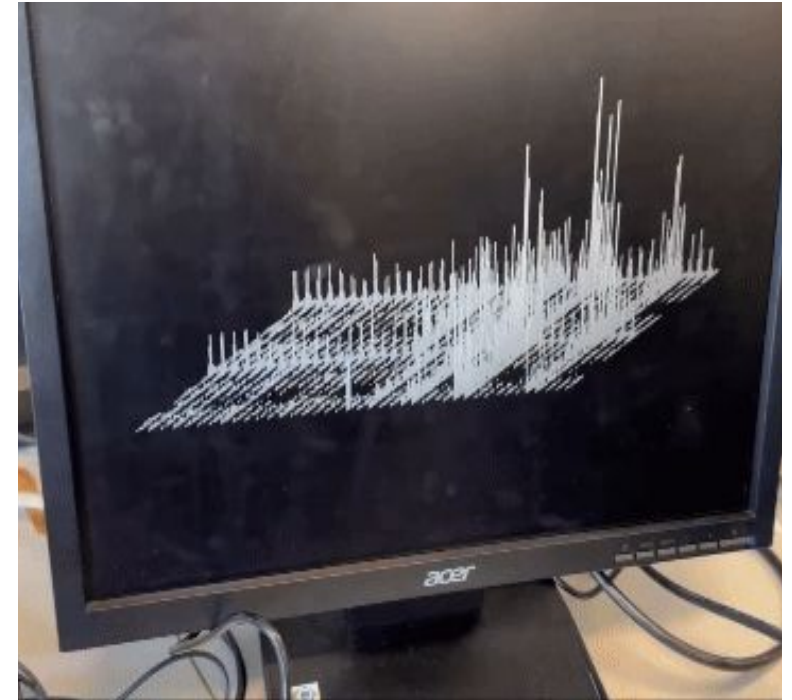
- Waveform Calculation

- Function $f(\text{CUR}, \text{Frames}, \text{ang})$ processes audio data and applies a rotational angle to generate the waveform.

Software

- **Displaying the Waveform**

- Clearing the Screen
 - Initial screen clear to reset previous data
- Setting Pixels:
 - Iteratively place pixels on the screen according to waveform data.
 - Use `set_pixel_axis` to position pixels and `set_background_color` to set intensity.
- Continuous Refresh
 - The screen is updated continuously as new audio data is processed.





Demo