

Data Parallel Programming with Repa

Stephen A. Edwards

Columbia University

Fall 2024



Iowa

Arrays

The Array Type: Shapes

Element Indices

Delayed Arrays and `computeS`
`fromFunction`

Example: Shortest Paths on a Dense Graph

The Floyd-Warshall Algorithm

Data-parallel, arithmetic-heavy algorithms over large arrays

REPA = REgular Parallel Arrays

<http://repa.ouroborus.net/>

Good array performance demands near-zero overhead per element

Strategies need lazy data structures; Par uses IVars.

Repa automatically parallelizes array computations

```
# stack.yaml
extra-deps:
- repa-3.4.1.4
```

```
# package.yaml
executables:
  repa-demo-exe:

  dependencies:
    - repa
```

Import the Repa module

```
import Data.Array.Repa as Repa
```

Some overlaps with the Prelude, so use, e.g., `Repa.map`, or specify exactly what you want to import, e.g.,

```
import Data.Array.Repa ( Array )
```

The Array Type: Includes the number of dimensions

data Array representation shape elements

Different ways to represent arrays in memory, e.g., U indicates unboxed

Haskell Types can't (yet) include numbers; the shape (number of dimensions) encoded as a *list of type constructors*

```
{-# LANGUAGE TypeOperators #-}
```

```
data Z          = Z          -- Zero-dimensional: a scalar
```

```
data tail :: head = tail :: head -- head: type of the dimension (Int)
```

```
type DIM0 = Z          -- Scalar
```

```
type DIM1 = DIM0 :: Int -- Vector
```

```
type DIM2 = DIM1 :: Int -- 2D Matrix
```

```
type DIM3 = DIM2 :: Int -- 3D Array
```

First Example

```
ghci> :set -XTypeOperators
ghci> :t fromListUnboxed
fromListUnboxed
  :: (Shape sh, Unbox a) => sh -> [a] -> Array U sh a
ghci> fromListUnboxed (Z :: 10) [1..10] :: Array U (Z :: Int) Int
AUnboxed (Z :: 10) [1,2,3,4,5,6,7,8,9,10]
```

Z and :: are both *type constructors* and *data constructors*

Z :: Int :: Int is the type for the shape of a 2-dimensional array

Z :: 5 :: 3 is the index for the element at row 5, column 3.

Matrices (2D Arrays)

Arrays are 1D vectors internally

```
ghci> fromListUnboxed (Z .. 3 .. 5) [1..15] :: Array U DIM2 Int
AUnboxed ((Z .. 3) .. 5) [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15]
```

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$$

Accessing a single element

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \end{bmatrix}$$

Column/row indices start from 0

```
ghci> :t (!)
(!) :: (Shape sh, Source r e) => Array r sh e -> sh -> e
ghci> let a = fromListUnboxed (Z::3::5) [1..15] ::Array U DIM2 Int
ghci> a ! (Z .. 1 .. 3)
9
ghci> :t toIndex
toIndex :: Shape sh => sh -> sh -> Int
ghci> toIndex (Z .. 3 .. 5 :: DIM2) (Z .. 1 .. 3 :: DIM2)
8
```


Querying the Shape of an Array

```
ghci> let a = fromListUnboxed (Z::3..5) [1..15] ::Array U DIM2 Int
ghci> :t extent
extent :: (Source r e, Shape sh) => Array r sh e -> sh
ghci> :t extent ar
extent ar :: DIM2
ghci> extent a                -- The shape
(Z .. 3) .. 5

ghci> :t rank                 -- Number of dimensions as an Int
rank :: Shape sh => sh -> Int
ghci> rank (extent a)
2

ghci> :t size                 -- Number of elements
size :: Shape sh => sh -> Int
ghci> size (extent a)
15
```

Operations on Arrays

Repa is designed to perform operations *across* arrays, not on single elements.
It uses compile-time array operation fusion

A D (Delayed) array hasn't yet been computed.

computeS can turn a Delayed into an Unboxed:

```
ghci> let a = fromListUnboxed (Z:.10) [1..10] :: Array U DIM1 Int
```

```
ghci> :t Repa.map          -- Produces an Array D ..
```

```
Repa.map :: (Shape sh, Source r a) =>  
          (a -> b) -> Array r sh a -> Array D sh b
```

```
ghci> :t Repa.map (+1) a
```

```
Repa.map (+1) a :: Array D DIM1 Int
```

```
ghci> computeS (Repa.map (+1) a) :: Array U DIM1 Int
```

```
AUnboxed (Z :. 10) [2,3,4,5,6,7,8,9,10,11]
```

fromFunction: Creating Delayed Arrays

```
ghci> :t fromFunction
fromFunction :: sh -> (sh -> a) -> Array D sh a

ghci> let a = fromFunction (Z:.10) (\(Z:.i) -> i * 10 :: Int)
ghci> :t a
a :: Array D (Z :. Int) Int

ghci> a ! (Z:.5)
50

ghci> computeS a :: Array U DIM1 Int
AUnboxed (Z :. 10) [0,10,20,30,40,50,60,70,80,90]
```

Roll-your-own Map over Arrays

```
ghci> let mymap f a = fromFunction (extent a) (\i -> f (a ! i))
ghci> :t mymap
mymap :: (Source r t, Shape sh) =>
        (t -> a) -> Array r sh t -> Array D sh a

ghci> let a = fromFunction (Z:.10) (\(Z:.i) -> i * 10 :: Int)
ghci> :t a
a :: Array D (Z :. Int) Int
ghci> :t mymap (+1) a
mymap (+1) a :: Array D (Z :. Int) Int
ghci> computeS (mymap (+1) a) :: Array U DIM1 Int
AUnboxed (Z :. 10) [1,11,21,31,41,51,61,71,81,91]
```

Floyd-Warshall Shortest Paths on a Dense Graph

In pseudocode:

```
shortestPath :: Graph -> Vertex -> Vertex -> Vertex -> Weight
shortestPath g i j 0 = weight g i j
shortestPath g i j k = min (shortestPath g i j (k-1))
                        (shortestPath g i k (k-1) +
                         shortestPath g k j (k-1))
```

An adjacency matrix is good for *dense* graphs

Sequential Implementation

```
type Weight = Int
type Graph r = Array r DIM2 Weight

shortestPaths :: Graph U -> Graph U
shortestPaths g0 = go g0 0
  where
    Z :: _ :: n = extent g0      -- Get # of vertices

    go !g !k | k == n      = g      -- Reached the end
              | otherwise = let    -- Compute new minimums
                  g' = computeS (fromFunction (Z:.n:.n) sp)
                  in go g' (k+1)  -- Increase k and repeat

    where sp (Z:.i:.j) =
      min (g ! (Z:.i:.j))           -- i → j
          (g ! (Z:.i:.k) + g ! (Z:.k:.j)) -- i → k → j
```

Making it Parallel

computeP runs in a monad to enforce sequential operation

```
ghci> :t computeP
computeP
  :: (Monad m, Source r2 e, Target r2 e, Load r1 sh e) =>
     Array r1 sh e -> m (Array r2 sh e)
```

Making it Parallel

```
import Data.Functor.Identity

shortestPaths :: Graph U -> Graph U
shortestPaths g0 = runIdentity $ go g0 0
  where
    Z :: _ :: n = extent g0

    go !g !k | k == n    = return g  -- Return in the monad
            | otherwise = do        -- We're in a monad
                g' <- computeP (fromFunction (Z::n::n) sp)
                go g' (k+1)

    where sp (Z::i::j) =
            min (g ! (Z::i::j))
                (g ! (Z::i::k) + g ! (Z::k::j))
```


Compiling it efficiently

```
executables:  
  fwdense1:  
    main: fwdense1.hs  
    source-dirs: app  
    ghc-options:  
      - -threaded          # Turn on parallelism  
      - -rtsopts           # Enable +RTS  
      - -O2                # Enable optimization  
      - -fllvm             # Enable LLVM backend  
    dependencies:  
      - repa
```

Statistics

Without -fllvm

Command-line	Time (s)
fwdense 500	2.808
fwdense1 500 +RTS -N1	3.110
fwdense1 500 +RTS -N2	1.690
fwdense1 500 +RTS -N4	1.160
fwdense1 500 +RTS -N5	1.180
fwdense1 500 +RTS -N6	1.041
fwdense1 500 +RTS -N7	0.911
fwdense1 500 +RTS -N8	1.321

With -fllvm

Command-line	Time (s)
fwdense 500	1.692
fwdense1 500 +RTS -N1	2.100
fwdense1 500 +RTS -N2	1.130
fwdense1 500 +RTS -N4	0.820
fwdense1 500 +RTS -N5	0.781
fwdense1 500 +RTS -N6	0.681
fwdense1 500 +RTS -N7	0.731
fwdense1 500 +RTS -N8	0.961