COMS W3261 Fall 2022 Handout 7a: Midterm Review

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1 Problem Set A: Regular Languages

Design finite automata for the following languages. You can give the DFA/NFA by their transition diagrams. You do not need to show that they are correct.
Note: In your transition diagrams, you can use shorthand notation on the labels of the edges. For example, you can label an edge by $\Sigma \setminus \{a\}$ to indicate that the transition takes place for all input symbols except a. Make sure to specify the starting state and the accepting states in your diagrams.

1. L is the language over the alphabet $\Sigma = \{0, 1, 2\}$ consisting of all strings that:
   - Every 0 is immediately followed by a 1, every 1 is immediately followed by a 2, and every 2 is immediately followed by a 0.
   - The string starts and ends with the same symbol.
   - The string must have length at least 1.

2. The set of strings over the alphabet $\Sigma = \{a, b, \ldots, z\}$ that contain at least one m between any two a's in the string; for example $abc, john, mama, american$ are in the language, but $papa, panamerican$ are not.

3. **Challenge:** The set of binary strings which represent in binary a number that is an integer multiple of 3 (leading zeros are allowed). For example, 00, 110 are in the language (they correspond to 0, 6 respectively), but 001, 101 are not (they correspond to 1, 5). Hint: Three states are enough. Think about what each additional symbol in a binary string does to the number; it might be helpful to write out some examples.

For the following problems, if a language L is given, prove that L is regular or prove that L is nonregular.

4. $L = \{ww \mid w \in \{0,1\}^*$ and w contains at least one 0 and at least one 1$\}$ over the alphabet $\Sigma = \{0,1\}$.

5. Prove that the union of a regular language L_1 and nonregular language L_2 such that $L_1 \cap L_2 = \emptyset$ results in a nonregular language.

6. $L = \{a^i b^j c^k \mid i + j = k\}$.

7. $L = \{0^k u 0^k \mid k \geq 1, u \in \Sigma^*\}$.

8. $L = \{0^k 1 u 0^k \mid k \geq 1, u \in \Sigma^*\}$.

9. $L = \{0^i 1^j \mid i, j \geq 0, i \neq j\}$.

10. $L = \{0^n 1^n \mid 0 \leq n \leq 3\}$.

11. **Challenge:** L is the language consisting of all strings of a's and b's with an equal number of occurrences of ab and ba as substrings. (The string $aabbbaa$ has one occurrence of each of the substrings ab and ba.)

2 Problem Set B: Context Free Languages

1. Show that the language

$$L = \{a^i b^j \mid i, j \geq 0, i \leq j \leq 3i\}$$

is context-free.

2. Give a context-free grammar that generates the language

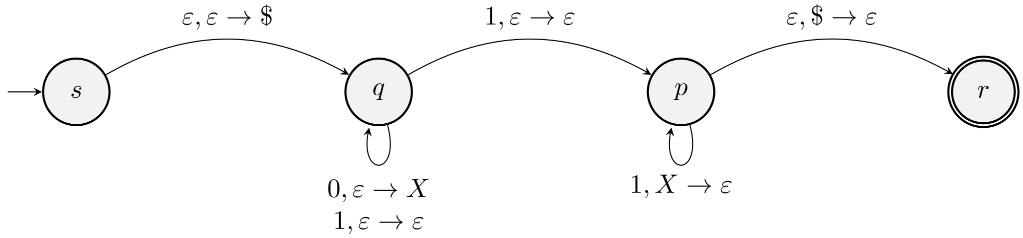
$$L = \{a^i b^j c^k : i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$$

Is your grammar ambiguous? Why or why not? (Consider the string $a^n b^n c^n$.)

3. Construct a pushdown automata for the language

$$L = \{a^n b^k c^n \mid n, k \geq 0\}$$

4. Describe the language that the following PDA recognizes:



5. **Challenge:** For any language L, let $\mathsf{SUFFIX}(A) = \{v \mid uv \in L \text{ for some string } u\}$. Show that the class of context-free languages is closed under the SUFFIX operation.

6. **Challenge:** Show that the complement of the language

$$L = \{ww \mid w \in \Sigma^*\}$$

is context-free.