COMS 3261 Handout 6A:
NFA and Regex Practice

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1 NFA

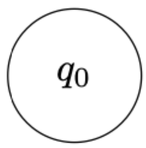
Problem 1. Draw an NFA that recognizes:

(a) All strings that contain 101.

(b) $L = \{w \in \{0,1\}^* | w \text{ has exactly two 0's or an even number of 1's}\}$

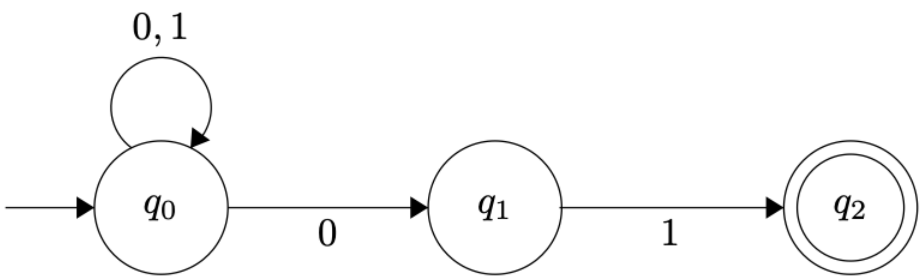
Problem 2.

(a) What is the language recognized by this NFA?



(b) What is the language recognized by this NFA?

Problem 3. Convert this NFA to an equivalent DFA using the subset construction:

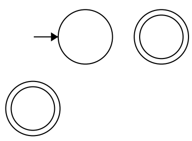


2 Closure properties of regular languages

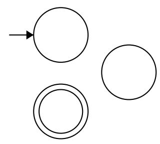
In class, we showed the class of regular languages is closed under concatenation, Kleene star and union, by using NFAs (in addition to previously showing closure under complement, intersection, and union, using DFAs). In this section, we briefly and informally review these operations and also prove other closure properties such as reversal, min, max and symmetric difference.

a) Closed under union

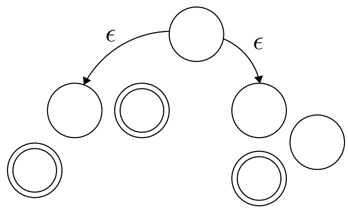
Recap. To construct the union of two languages, we add a new start state and ϵ-transition to the old start states.



L_1



L_2

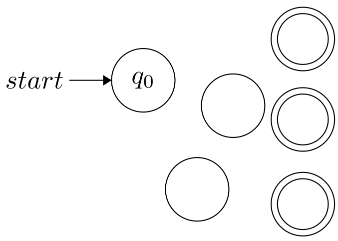


$L_1 \cup L_2$

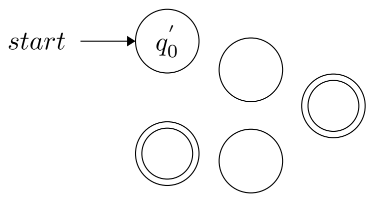
Problem 4. $L = \{w \in \{0,1\}^* | w$ has an even number of 0's, or w does not have two consecutive $1's\}$. In homework 1 q2a, we expressed L as the union of two regular languages. Using the DFA constructed for each language, now draw an NFA that recognizes this union.

b) Closed under concatenation

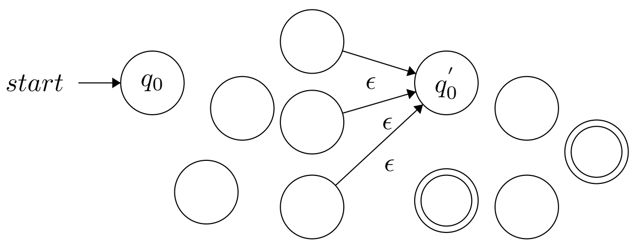
Recap. To construct the concatenation of L_1 and L_2, we add ϵ transitions from the accepting states of L_1 to the start state of L_2



L_1



L_2

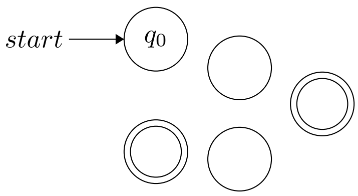


$L_1 \cdot L_2$

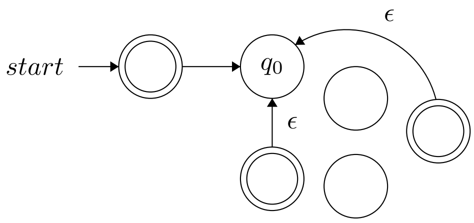
Problem 5. Draw an NFA for L_1, L_2, and $L_1 \circ L_2$.
$L_1 = \{w \in \{0,1\}^* | w$ has an odd number of $1's$ or exactly two 0s$\}$.
$L_2 = \{w \in \{0,1\}^* | w$ ends with 101$\}$.

c) Closed under Kleene star

Recap.



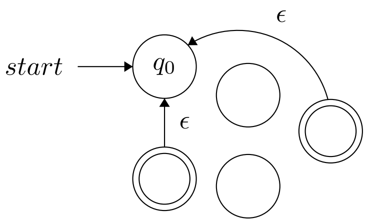
L



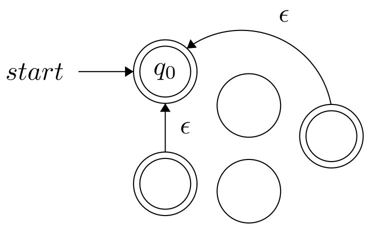
L^*

In class we saw multiple incorrect attempts to represent the Kleene star operation on the state diagram of regular languages (these attempts work for some specific cases, but not always).

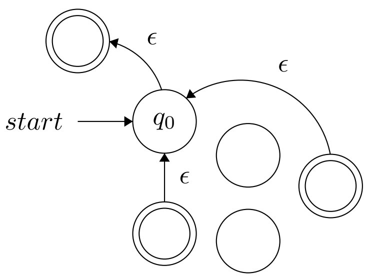
Problem 6. For each of the following incorrect construction of Kleene star operation, suggest a counter example to show why the attempt is incorrect.



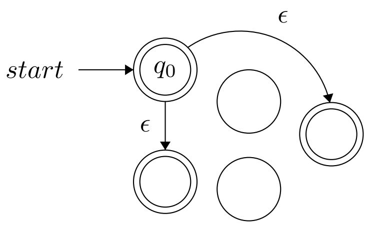
Bad attempt 1



Bad attempt 2



Bad attempt 3



Bad attempt 4

d) More operations

Problem 7. Let L be a regular language. Prove that the following operations applied to L also results in regular languages.

(a) $L^R := \{w^R \mid w \in L\}$
Recall that R is the reversal operation that when applied to a string $w = w_1 w_2 \ldots w_{n-1} w_n$ results in $w^R = w_n w_{n-1} \ldots w_2 w_1$.

(b) $\min(L) = \{w \in \Sigma^* \mid w$ is in L and no proper prefix of w is in L$\}$
A proper prefix of a string w is a prefix of w that is not equal to w

3 Regular Expressions

Problem 8. Describe in words the language expressed by these regular expressions:

(a) 0^*1^*

(b) $(01)^*$

(c) $(0^*1^*)^*$

(d) $(0 \cup 1)^*$

(e) $0^*(10^*1)^*0^*$

Problem 9. Construct regular expressions for the following languages.

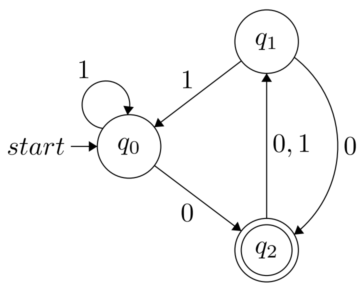
(a) All binary strings that start with 10

(b) All binary strings that have exactly two 0s

(c) All binary strings with odd number of 0s

(d) All strings over $\{a, b, c\}$ that do not contain the substring abc. Challenging!

Problem 10. Convert the following DFA to an equivalent regular expression, using the construction we saw in class going through GNFAs.



Problem 11. Convert the following regular expression into an equivalent NFA

$$(01)^*(10^* \cup 0)$$