COMS 4995 Lecture 5: Convolutional Neural Networks & Image Classification, Part II

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2-D Convolution

2-D convolution is defined analogously to 1-D convolution.

If A and B are two 2-D arrays, then:

$$(A * B)_{ij} = \sum_s \sum_t A_{st} B_{i-s,j-t}.$$

2-D Convolution

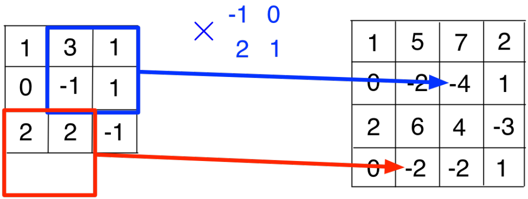
Method 1: Translate-and-Scale

$$
\begin{array}{|c|c|c|}\hline 1 & 3 & 1 \\ \hline 0 & -1 & 1 \\ \hline 2 & 2 & -1 \\ \hline \end{array}
\ast
\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 0 & -1 \\ \hline \end{array}
=
\begin{array}{l}
1 \times \begin{array}{|c|c|c|c|}\hline 1 & 3 & 1 & \\ \hline 0 & -1 & 1 & \\ \hline 2 & 2 & -1 & \\ \hline & & & \\ \hline \end{array} \\[1em]
+\ 2 \times \begin{array}{|c|c|c|c|}\hline & 1 & 3 & 1 \\ \hline & 0 & -1 & 1 \\ \hline & 2 & 2 & -1 \\ \hline & & & \\ \hline \end{array} \\[1em]
+ -1 \times \begin{array}{|c|c|c|c|}\hline & & & \\ \hline & 1 & 3 & 1 \\ \hline & 0 & -1 & 1 \\ \hline & 2 & 2 & -1 \\ \hline \end{array}
\end{array}
=
\begin{array}{|c|c|c|c|}\hline 1 & 5 & 7 & 2 \\ \hline 0 & -2 & -4 & 1 \\ \hline 2 & 6 & 4 & -3 \\ \hline 0 & -2 & -2 & 1 \\ \hline \end{array}
$$

2-D Convolution

Method 2: Flip-and-Filter

$$\begin{array}{|c|c|c|}\hline 1 & 3 & 1 \\ \hline 0 & -1 & 1 \\ \hline 2 & 2 & -1 \\ \hline \end{array} \ast \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 0 & -1 \\ \hline \end{array}$$



2-D Convolution

The thing we convolve by is called a kernel, or filter.

What does this convolution kernel do?

$*$

0	1	0
1	4	1
0	1	0

2-D Convolution

The thing we convolve by is called a kernel, or filter.

What does this convolution kernel do?

$*$

0	1	0
1	4	1
0	1	0

Answer: Blur

Note: We call the resulting image an "activation map" by the kernel

2-D Convolution

What does this convolution kernel do?

$*$

0	-1	0
-1	8	-1
0	-1	0

2-D Convolution

What does this convolution kernel do?

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Answer: Sharpen

2-D Convolution

What does this convolution kernel do?

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-1	4	-1
0	-1	0

2-D Convolution

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Answer: Edge Detection

2-D Convolution

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$*$

1	0	-1
2	0	-2
1	0	-1

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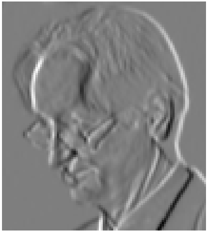
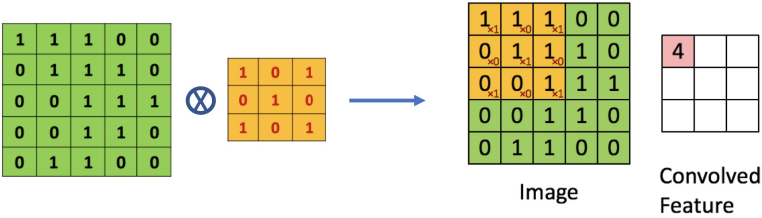
1	0	-1



Answer: Stronger Edge Detection

Example: A closer look at convolution



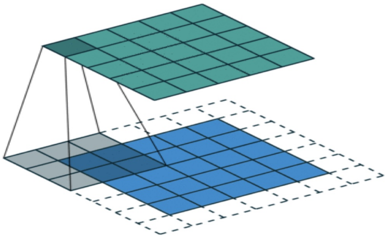
Input: 5 x 5

Kernel: 3 x 3

Output: 3x 3

$$3 = (5 - 3) + 1$$

Example: A closer look at convolution with padding



Input: 5 x 5

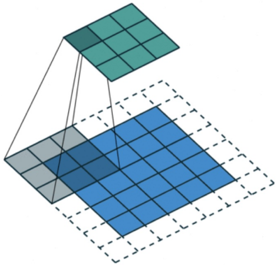
Kernel: 3 x 3

Padding: 1

Output: 5 x 5

5 = (5 – 3 + 2 * 1) + 1

Example: A closer look at convolution with stride

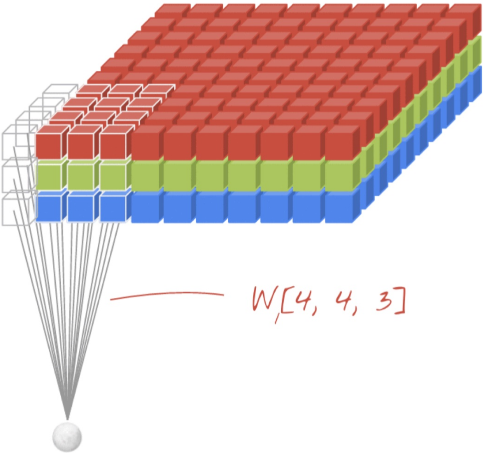


Input: 5 x 5
Kernel: 3 x 3
Padding: 1
Stride = 2

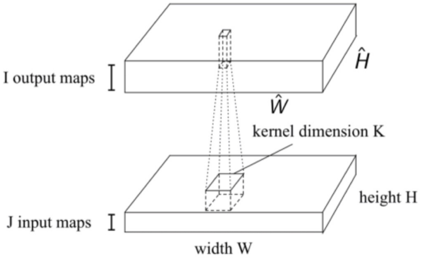
Output: 3 x 3

$$3 = (5 - 3 + 2 * 1)/2 + 1$$

Example: A closer look at convolution with high dimensional inputs



Summary of Convolution Layer



- Input: An array of size $W \times H \times J$
- Hyper-parameters:
  - Number of filters: M
  - Size of filters: K
  - the stride: S
  - Number of zero-padding: P
- Output: Feature maps of size $\hat{W} \times \hat{H} \times I$
  - $\hat{W} = (W - K + 2P)/S + 1$
  - $\hat{H} = (H - K + 2P)/S + 1$
  - $I = M$

Size of a Conv Net

- Ways to measure the size of a network:
  - **Number of units.** This is important because

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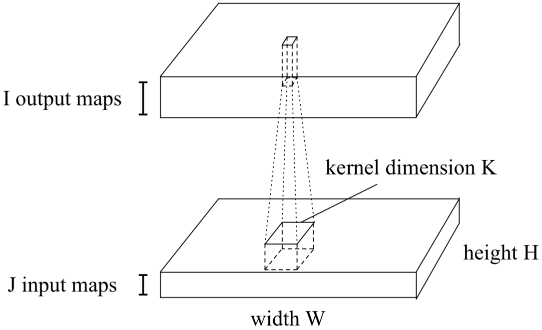
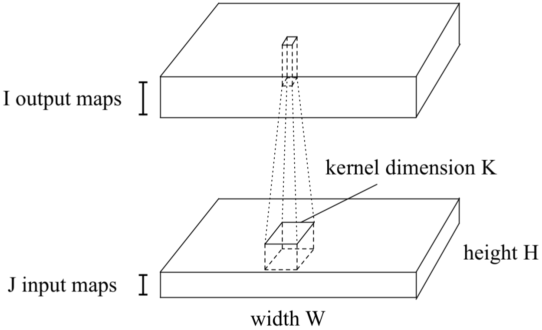
  - **Number of connections.** This is important because there are approximately 3 add-multiply operations per connection (1 for the forward pass, 2 for the backward pass).
- We saw that a fully connected layer with M input units and N output units has MN connections and MN weights.
- The story for conv nets is more complicated.

Size of a Conv Net

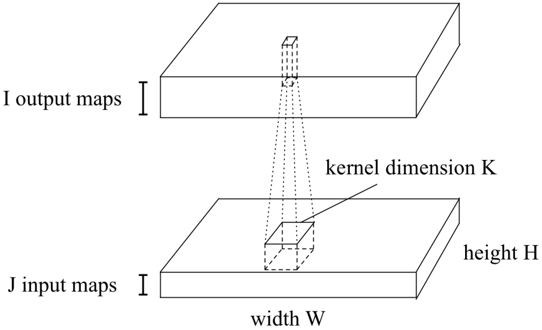


Size of a Conv Net



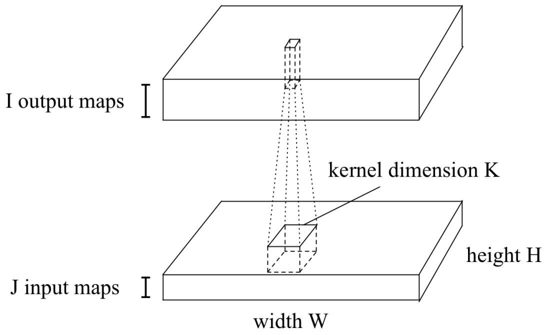
	fully connected layer	convolution layer
# output units		

Size of a Conv Net



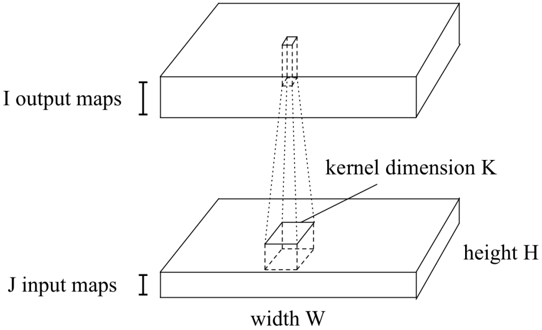
	fully connected layer	convolution layer
# output units	*WHI*	*WHI*

Size of a Conv Net



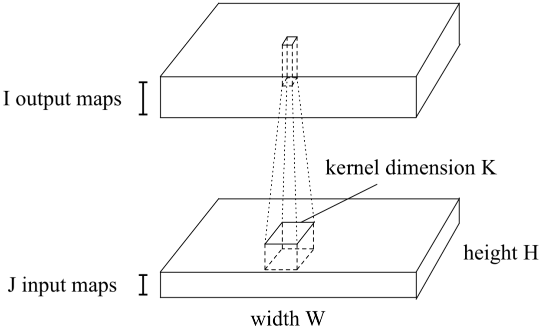
	fully connected layer	**convolution layer**
# output units	*WHI*	*WHI*
# weights		

Size of a Conv Net



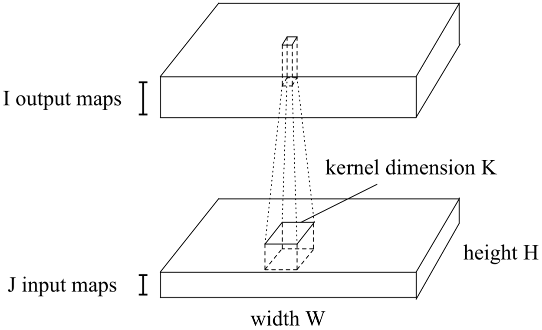
	fully connected layer	convolution layer
# output units	WHI	WHI
# weights	W^2H^2IJ	

Size of a Conv Net



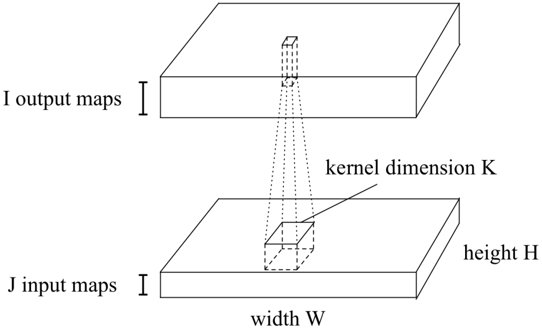
	fully connected layer	convolution layer
# output units	WHI	WHI
# weights	W^2H^2IJ	K^2IJ

Size of a Conv Net



	fully connected layer	**convolution layer**
# output units	WHI	WHI
# weights	W^2H^2IJ	K^2IJ
# connections		

Size of a Conv Net



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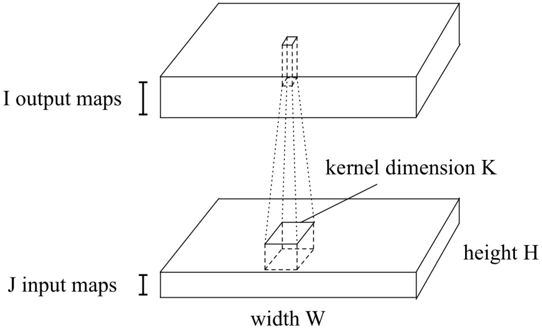
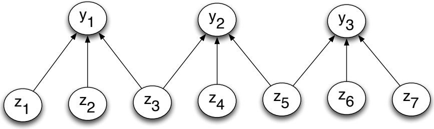


	fully connected layer	convolution layer
# output units	WHI	WHI
# weights	W^2H^2IJ	K^2IJ
# connections	W^2H^2IJ	WHK^2IJ

Pooling layers

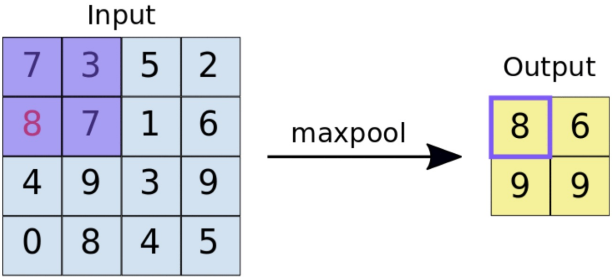
The other type of layer in a pooling layer. These layers reduce the size of the representation and build in invariance to small transformations.



Most commonly, we use max-pooling, which computes the maximum value of the units in a pooling group:

$$y_i = \max_{j \text{ in pooling group}} z_j$$

Pooling layer

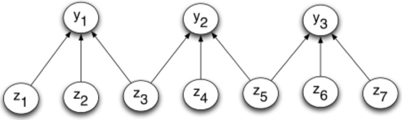


Input: 4 x 4
Kernel: 2 x 2
Stride: 2

Output: 2 x 2

$2 = (4 - 2) / 2 + 1$

Summary: Pooling layer



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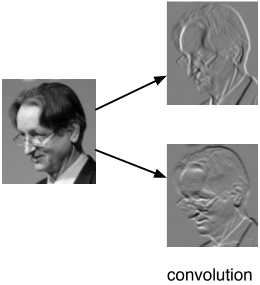
- Hyper-parameters:
  - Size of filters: K
  - the stride: S
- Output: Feature maps of size $\hat{W} \times \hat{H} \times I$
  - $\hat{W} = (W - K)/S + 1$
  - $\hat{H} = (H - K)/S + 1$
  - $I = J$

Convolutional networks

Let's finally turn to convolutional networks. These have two kinds of layers: detection layers (or convolution layers), and pooling layers.

The convolution layer has a set of filters. Its output is a set of feature maps, each one obtained by convolving the image with a filter.



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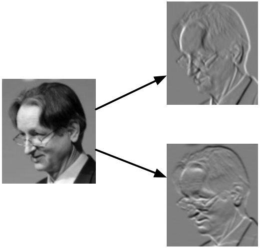
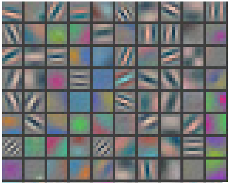


convolution

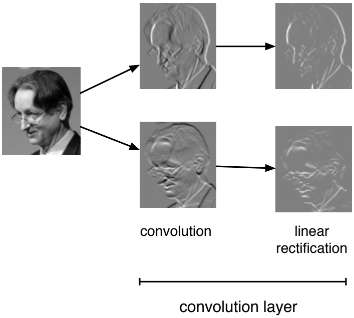
Example first-layer filters



(Zeiler and Fergus, 2013, Visualizing and understanding convolutional networks)

Convolutional networks

It's common to apply a linear rectification nonlinearity: $y_i = \max(z_i, 0)$



Why might we do this?

Convolutional networks

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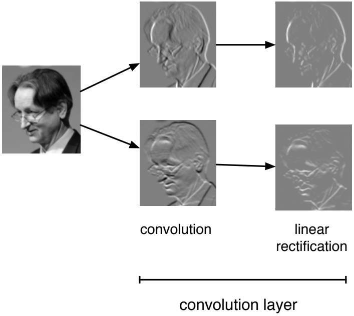
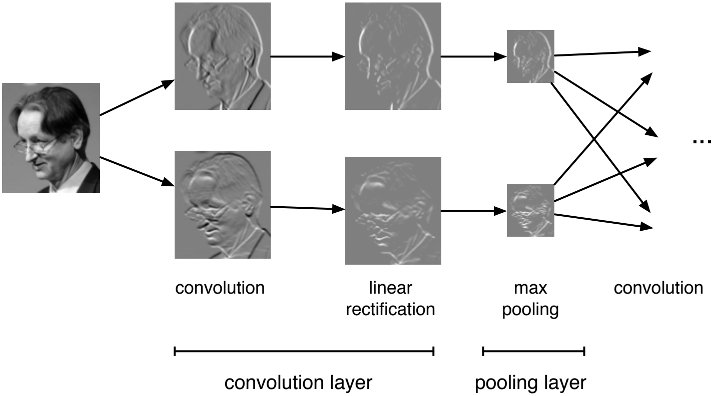


Why might we do this?

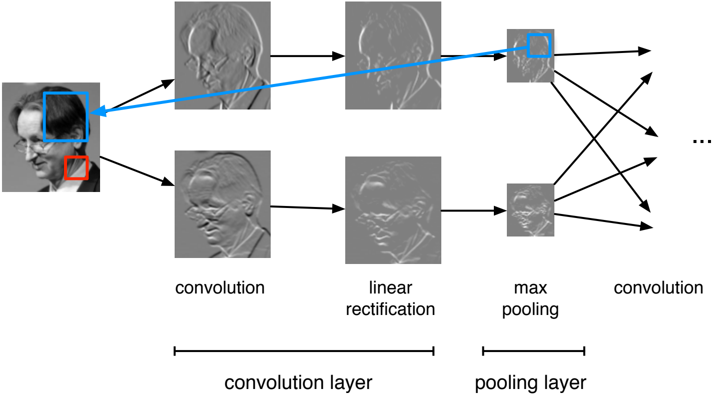
- Convolution is a linear operation. Therefore, we need a nonlinearity, otherwise 2 convolution layers would be no more powerful than 1.
- Two edges in opposite directions shouldn't cancel

Convolutional networks



Convolutional networks

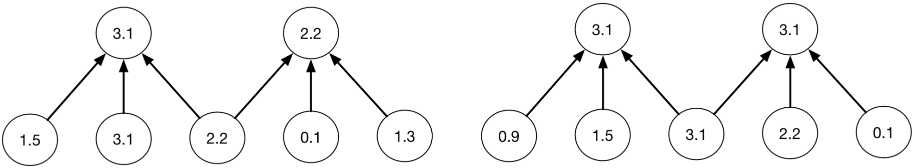
Because of pooling, higher-layer filters can cover a larger region of the input than equal-sized filters in the lower layers.



Equivariance and Invariance

We said the network's responses should be robust to translations of the input. But this can mean two different things.

- Convolution layers are equivariant: if you translate the inputs, the outputs are translated by the same amount.
- We'd like the network's predictions to be invariant: if you translate the inputs, the prediction should not change.
- Pooling layers provide invariance to small translations.



Convolution Layers

Each layer consists of several feature maps, or channels each of which is an array.

- If the input layer represents a grayscale image, it consists of one channel. If it represents a color image, it consists of three channels.

Each unit is connected to each unit within its receptive field in the previous layer. This includes *all* of the previous layer's feature maps.

Convolution Layers

For simplicity, focus on 1-D signals (e.g. audio waveforms). Suppose the convolution layer's input has J feature maps and its output has I feature maps. Let t index the locations. Suppose the convolution kernels have radius R, i.e. dimension $K = 2R + 1$.

Each unit in a convolution layer receives inputs from all the units in its receptive field in the previous layer:

$$ {}_{i,t} = \sum_{j=1}^{J} \sum_{\tau=-R}^{R} w_{i,j,\tau} {}_{j,t+\tau}. $$

In terms of convolution,

$$ {}_{i} = \sum_{j} {}_{j} * \mathrm{flip}({}_{i,j}). $$

Object recognition

- Object recognition is the task of identifying which object category is present in an image.
- It's challenging because objects can differ widely in position, size, shape, appearance, etc., and we have to deal with occlusions, lighting changes, etc.
- Why we care about it
  - Direct applications to image search
  - Closely related to object detection, the task of locating all instances of an object in an image
    - E.g., a self-driving car detecting pedestrians or stop signs
- For the past decade, all of the best object recognizers have been various kinds of conv nets.

Datasets

- In order to train and evaluate a machine learning system, we need to collect a dataset. The design of the dataset can have major implications.
- Some questions to consider:
  - Which categories to include?
  - Where should the images come from?
  - How many images to collect?
  - How to normalize (preprocess) the images?

Image Classification

- Conv nets are just one of many possible approaches to image classification. However, they have been by far the most successful for the last decade.
- Biggest image classification "advances" of the last two decades
  - Datasets have gotten much larger (because of digital cameras and the Internet)
  - Computers got much faster
    - Graphics processing units (GPUs) turned out to be really good at training big neural nets; they're generally about 30 times faster than CPUs.
  - As a result, we could fit bigger and bigger neural nets.

MNIST Dataset

- MNIST dataset of handwritten digits
  - **Categories:** 10 digit classes
  - **Source:** Scans of handwritten zip codes from envelopes
  - **Size:** 60,000 training images and 10,000 test images, grayscale, of size 28×28
  - **Normalization:** centered within in the image, scaled to a consistent size
    - The assumption is that the digit recognizer would be part of a larger pipeline that segments and normalizes images.
- In 1998, Yann LeCun and colleagues built a conv net called LeNet which was able to classify digits with 98.9% test accuracy.
  - It was good enough to be used in a system for automatically reading numbers on checks.

ImageNet

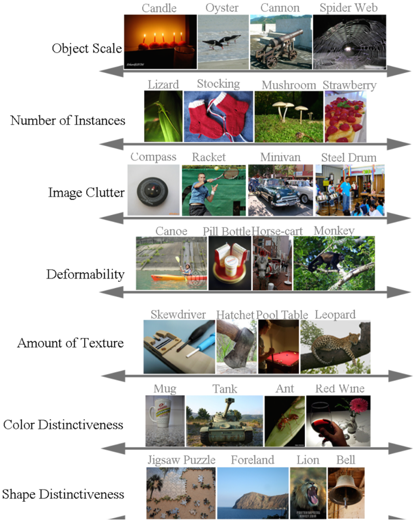
ImageNet is the modern object recognition benchmark dataset. It was introduced in 2009, and has led to amazing progress in object recognition since then.

ImageNet

- Used for the ImageNet Large Scale Visual Recognition Challenge (ILSVRC), an annual benchmark competition for object recognition algorithms
- Design decisions
  - **Categories:** Taken from a lexical database called WordNet
    - WordNet consists of "synsets", or sets of synonymous words
    - They tried to use as many of these as possible; almost 22,000 as of 2010
    - Of these, they chose the 1000 most common for the ILSVRC
    - The categories are really specific, e.g. hundreds of kinds of dogs
  - **Size:** 1.2 million full-sized images for the ILSVRC
  - **Source:** Results from image search engines, hand-labeled by Mechanical Turkers
    - Labeling such specific categories was challenging; annotators had to be given the WordNet hierarchy, Wikipedia, etc.
  - **Normalization:** none, although the contestants are free to do preprocessing

ImageNet

Images and object categories vary on a lot of dimensions



Russakovsky et al.

ImageNet

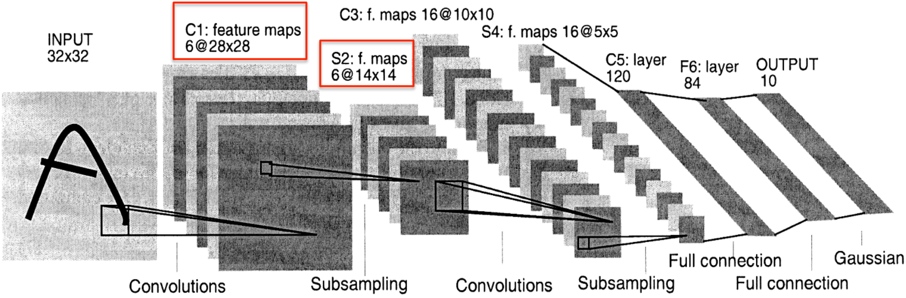
Size on disk:

MNIST
60 MB

ImageNet
50 GB

LeNet

Here's the LeNet architecture, which was applied to handwritten digit recognition on MNIST in 1998:



Size of a Conv Net: LeNet

Sizes of layers in LeNet:

Layer	Type	# units	# connections	# weights
C1	convolution	4704	117,600	150
S2	pooling	1176	4704	0
C3	convolution	1600	240,000	2400
S4	pooling	400	1600	0
F5	fully connected	120	48,000	48,000
F6	fully connected	84	10,080	10,080
output	fully connected	10	840	840

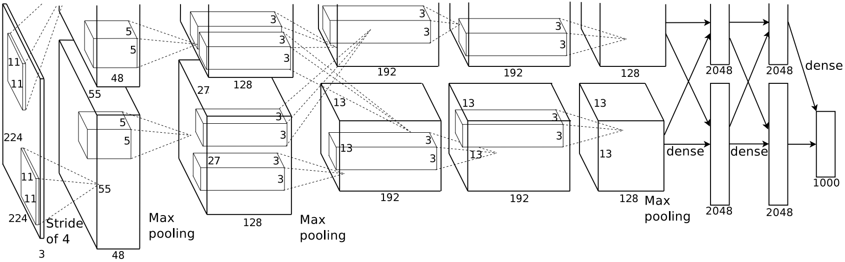
Conclusions?

Size of a Conv Net

- Rules of thumb:
  - Most of the units and connections are in the convolution layers.
  - Most of the weights are in the fully connected layers.
- If you try to make layers larger, you'll run up against various resource limitations (i.e. computation time, memory)
- Conv nets have gotten a LOT larger since 1998!

AlexNet

- AlexNet, 2012. 8 weight layers. 16.4% top-5 error (i.e. the network gets 5 tries to guess the right category).



(Krizhevsky et al., 2012)

- They used lots of tricks we've covered in this course (ReLU units, weight decay, data augmentation, SGD with momentum, dropout)
- AlexNet's stunning performance on the ILSVRC is what set off the deep learning boom of the last 6 years.

Size of a Conv Net: Comparison

	LeNet (1989)	LeNet (1998)	AlexNet (2012)
classification task	digits	digits	objects
categories	10	10	1,000
image size	16×16	28×28	$256 \times 256 \times 3$
training examples	7,291	60,000	1.2 million
units	1,256	8,084	658,000
parameters	9,760	60,000	60 million
connections	65,000	344,000	652 million

GoogLeNet

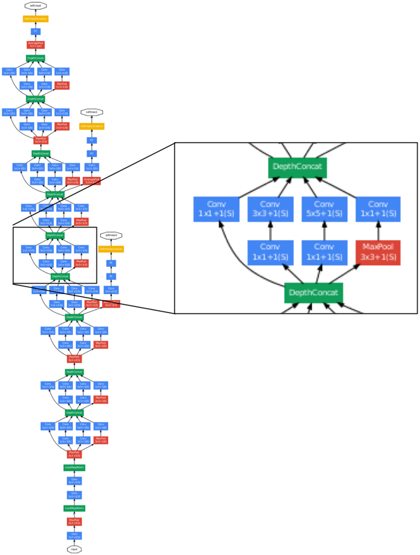
GoogLeNet, 2014.

22 weight layers

Fully convolutional (no fully connected layers)

Convolutions are broken down into a bunch of smaller convolutions

6.6% test error on ImageNet



GoogLeNet

- They were really aggressive about cutting the number of parameters.
  - Motivation: train the network on a large cluster, run it on a cell phone
    - Memory at test time is the big constraint.
    - Having lots of units is OK, since the activations only need to be stored at training time (for backpropagation).
    - Parameters need to be stored both at training and test time, so these are the memory bottleneck.
  - How they did it
    - No fully connected layers (remember, these have most of the weights)
    - Break down convolutions into multiple smaller convolutions (since this requires fewer parameters total)
  - GoogLeNet has "only" 2 million parameters, compared with 60 million for AlexNet
  - This turned out to improve generalization as well. (Overfitting can still be a problem, even with over a million images!)

Classification

ImageNet results over the years. Note that errors are top-5 errors (the network gets to make 5 guesses).

Year	**Model**	**Top-5 error**
2010	Hand-designed descriptors + SVM	28.2%
2011	Compressed Fisher Vectors + SVM	25.8%
2012	AlexNet	16.4%
2013	a variant of AlexNet	11.7%
2014	GoogLeNet	6.6%
2015	deep residual nets	4.5%

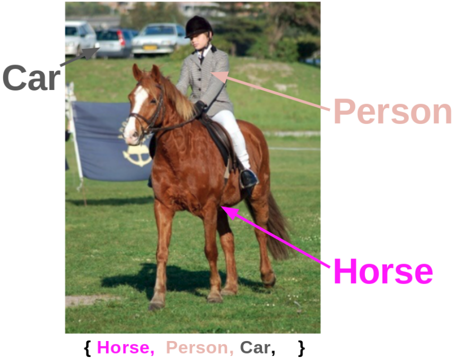
We'll cover deep residual nets later in the course, since they require an idea we haven't covered yet.

Human-performance is around 5.1%.

They stopped running the object recognition competition because the performance is already so good.

Beyond Classification

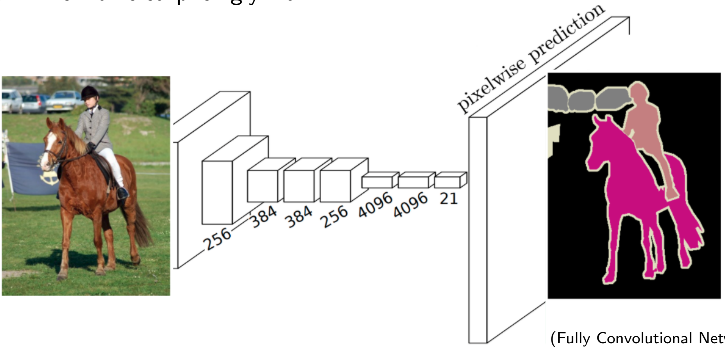
- The classification nets map the entire input image to a pre-defined class categories.
- But there are more than just class labels in an image.
  - where is the foreground object? how many? what is in the background?



(PASCAL VOC 2012)

Semantic Segmentation

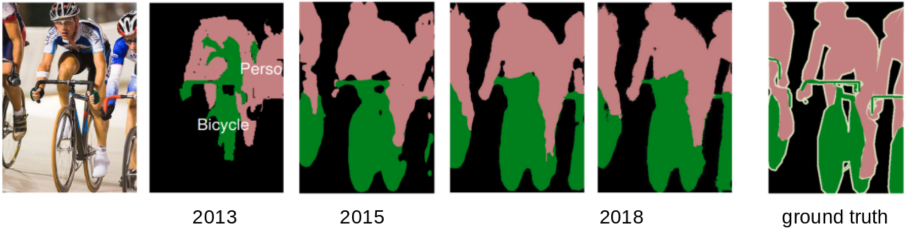
- Semantic segmentation, a natural extention of classification, focuses on making dense classification of class labels for every pixel.
- It is an important step towards complete scene understanding in compter vision.
  - Semantic segmentation is a stepping stone for many of the high-level vision tasks, such as object detection, Visual Question Answering (VQA).
- A naive approach is to adapt the existing object classification conv nets for each pixel. This works surprisingly well.



(Fully Convolutional Networks, 2015)

Semantic Segmentation

- After the success of CNN classifiers, segmentation models quickly moved away from hand-craft features and pipelines but instead use CNN as the main structure.
- Pre-trained ImageNet classification network serves as a building block for all the state-of-the-art CNN-based segmentation models.



from left to wright (Li, et. al., (CSI), CVPR, 2013; Long, et. al., (FCN), CVPR 2015; Chen et. al., (DeepLab), PAMI 2018)